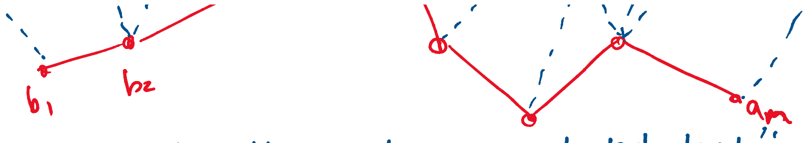


" walking a dog with shortest leash "

Thm Assuming SETH,
no $O(n^{2-\delta})$ alg'm for these problems

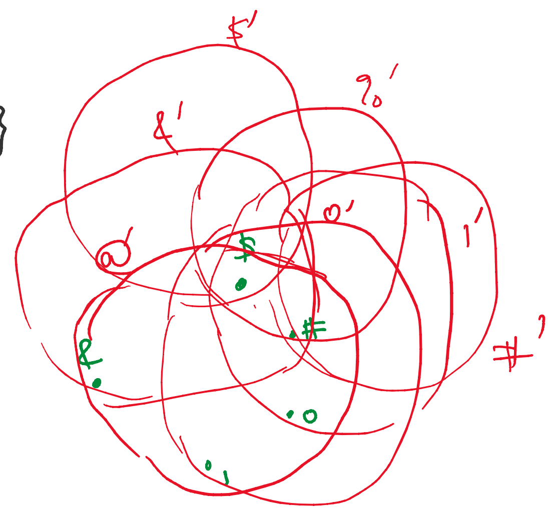
History: by Bringmann '14 for Frechet dist. (discrete & continuous)
Backurs, Indyk '15 for edit dist.
Abboud, Backurs, Vassilevska '15 } for LCS
Bringmann, Künnemann '15

Reduce OV → Discrete Frechet (Bringmann '14)

Suppose disc Frechet could be solved in $O(n^{2-\delta})$ time.
Given vectors $a_1, \dots, a_n, b_1, \dots, b_n \in \{0,1\}^d$.
define alphabet $\Sigma = \{0, 1, \&, \$, \#, 0', 1', \&', \$', \#', \%', @'\}$

$d(\cdot,\cdot)$	$0'$	$1'$	$\&'$	$\%'$	$\$'$	$\#'$	$@'$
0	$<r$	$<r$					$<r$
1	$<r$						$<r$
&			$<r$				$<r$
$	$<r$	$<r$	$<r$	$<r$	$<r$		$<r$
#	$<r$	$<r$	$<r$	$<r$	~~$<r$~~	$<r$	$<r$

all other $>r$



Define strings $A = \$ f(a_1) \# \$ f(a_2) \# \quad \cdots \quad \$ f(a_n) \#$
$B = @' \$' g(b_1) \%' g(b_2) \%' \quad \cdots \quad \%' g(b_n) \#' @'$ ⟸ $O(dn)$

where $f(a) = a[1] \, \& \, a[2] \, \& \quad \cdots \quad \& \, a[d]$
$g(b) = b[1]' \, \&' \, b[2]' \, \&' \quad \cdots \quad \&' \, b[d]'$

$O((dn)^{2-\delta})$

Claim disc Frechet dist $< r$ ⟺ $\exists$ orth pair.

Pf: (⟸) Suppose $\exists i,j$ $a_i \cdot b_j = 0$.

$\$ f(a_i) \#$

$A = \$$

$B = @' \; \$' \quad \%' g(b_j) \%' \; - - - - \; \#' @'$

then Frechet dist $< r$.

($\Longrightarrow$) Suppose Frechet dist $< r$.

$A = \quad \$\ f(a_i) \quad \#$

$B = @'\ \$' \quad g(b_j) \quad \#'@'$

$\Longrightarrow \quad a_i \cdot b_j = 0$ $\square$

Rmks:

- same pf implies hardness of c-approx for same const c
- edit dist / LCS / DTW "similar" but more complicated
- Abboud et al. '16: holds for much weaker version of SETH (for Circuit-SAT with sublinear depth)

Cond. Lower Bds via 3SUM

3SUM Problem: Given set S of n numbers, decide $\exists a, b, c \in S$ s.t. $a+b+c=0$

(3-Set version: given A, B, C, $\exists a \in A, b \in B, c \in C$ s.t. $\cancel{a+b+c=0}$ $a+b=c$)

(all vers. are equiv)

Conjecture: no $O(n^{2-\delta})$-time alg'm for 3SUM
... for ints)

Conjecture no $O(n^{2-\delta})$-time alg'm for 3SUM
(for reals, or for ints)
(strongest: for ints in $[n^2]$).

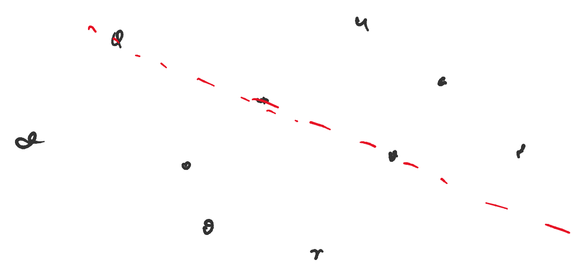
History: Gajentaan-Overmars '93 in comp geometry
(predates SETH, APSP Conj, etc.)

(problems reducible from 3SUM
called 3SUM-hard)

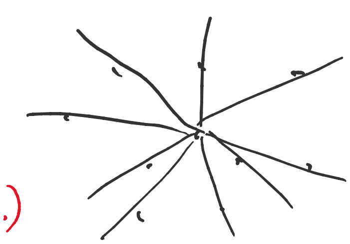
Exs of 3SUM-Hard Problems in Comp. Geometry

3-Collinear-Pts (affine degeneracy testing)

Given set P of n pts in 2D,
decide $\exists$ 3 pts of P lying on a common line

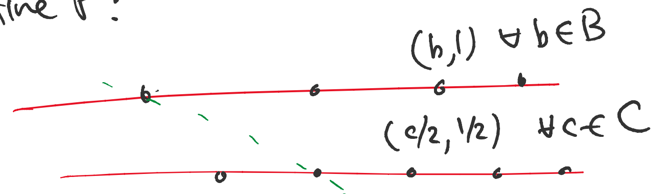


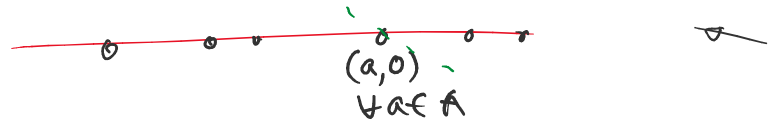
trivial: $O(n^3)$
best known alg: $O(n^2)$



3SUM (3 sets vers.) $\longrightarrow$ **3-Collinear-Pts** (3 sets vers.):

given sets A, B, C of n numbers,
define P:





$(a,0),\ (b,1),\ (\frac{c}{2},\frac{1}{2})$ collinear

$\iff a+b=c$

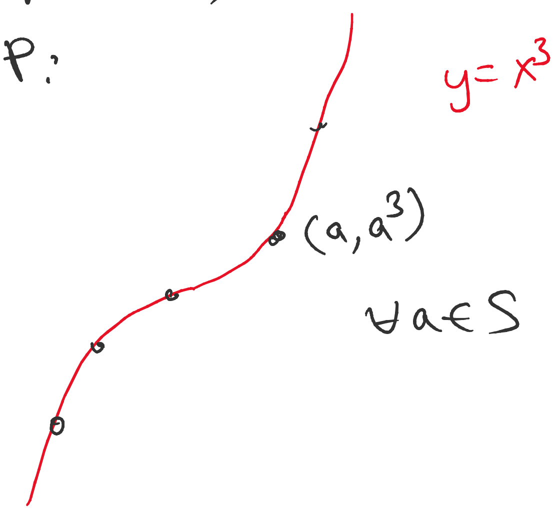
(1 set)

(1 set vers.)

3SUM → 3-Collinear-Pts:

given S, of n numbers,

define P:



$(a,a^3),\ (b,b^3),\ (c,c^3)$ collinear

$\iff a+b+c=0$.

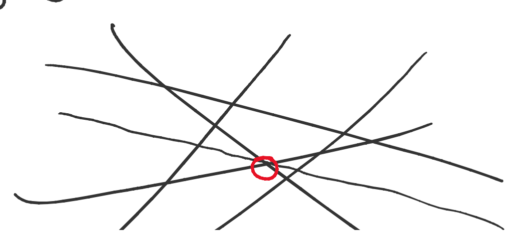
(extends to d dims: $x \to (x, x^2, x^3, \ldots, x^{d-1}, x^{d+1})$

weird moment curve

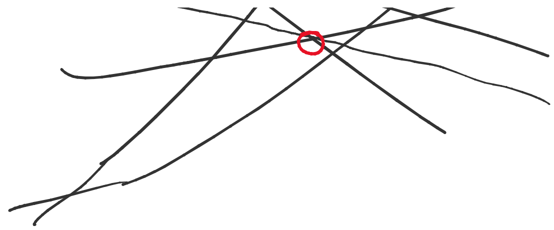
by Jeff E.)

3-Concurrent-Lines:

Given n lines in 2D, decide ∃ 3 lines that intersect at common pt

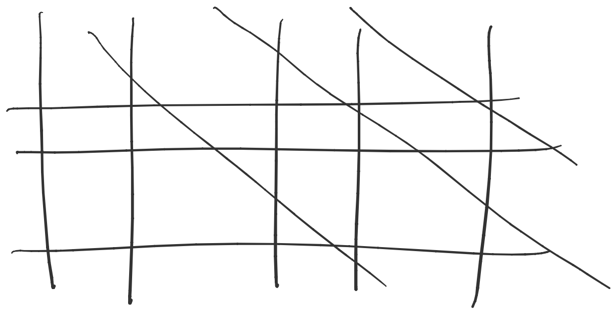


(3-Concurrent-Lines $\longleftrightarrow$ 3-Collinear-Pts)



$\longleftrightarrow$ 3-Collinear

3SUM $\xrightarrow{\text{(3-set vers.)}}$ 3-Concurrent-Lines:



$y=b$

$\forall b \in B$

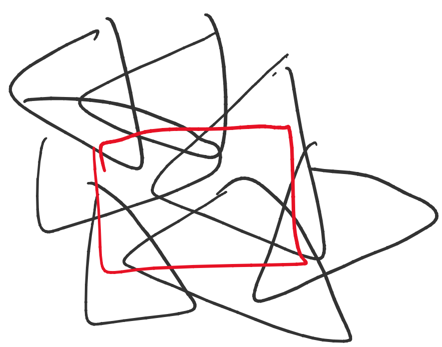
$x=a$

$\forall a \in A$

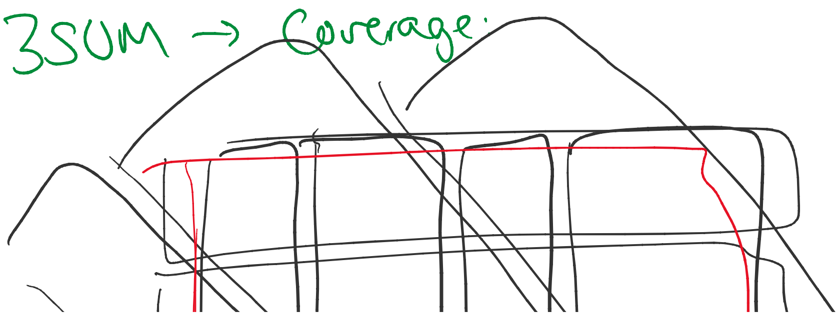
$x+y=c$

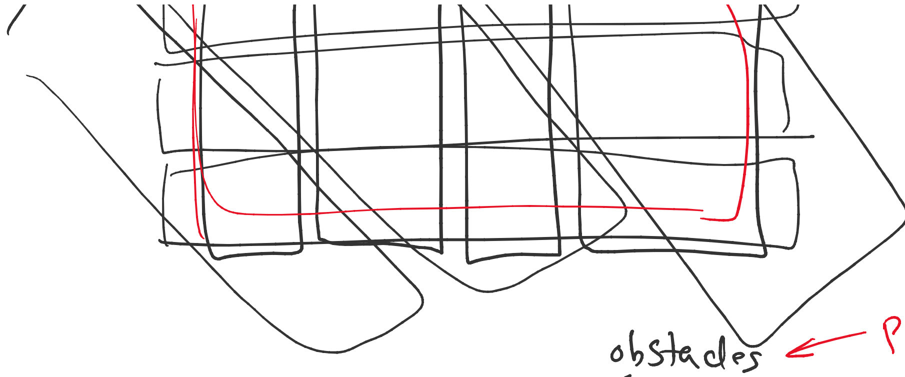
$\forall c \in C.$

Coverage: Given n objects (e.g. triangles) in 2D, decide whether union covers a region (e.g. rectangle)



3SUM → Coverage:





obstacles ← polygons

Motion Planning: Given n objects in 2D
& robot

decide $\exists$ way to move robot
from one position to another

3SUM → Motion Planning: