August 24, 2021 10:15 AM

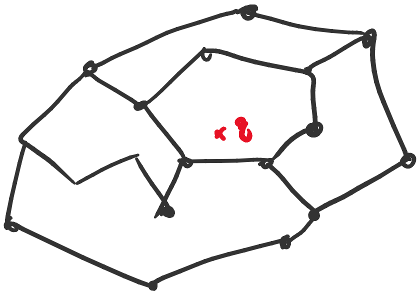
What is Comp. Geometry?

- alg'ms for geom. probs
- lots of appl'ns
- SoCG

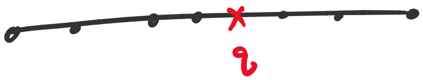
Possible Topics

- geom. data structures

point location



in 1D:

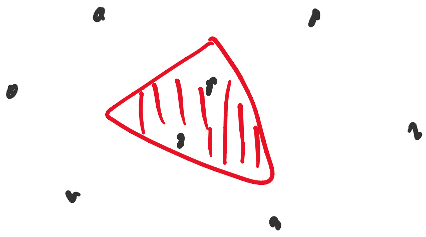


$O(n)$ space by sorting
$O(\log n)$ query time

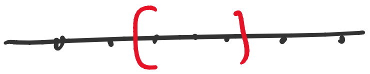
in 2D? more challenging

dynamic?

range search



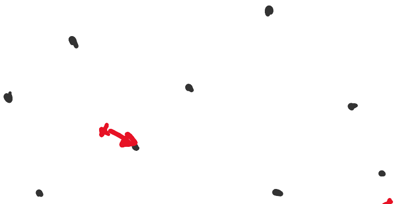
in 1D,



$O(n)$ space
$O(\log n)$ time to count

in 2D ?

nearest neighbor search

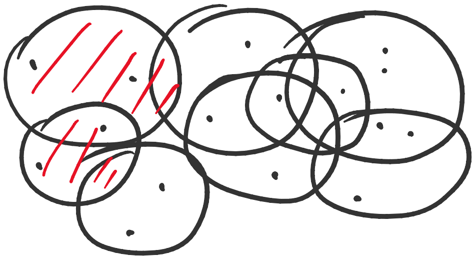


(dynamic? $O(\log^{\cancel{6}\,\cancel{5}\,4} n)$ update
$O(\log^2 n)$ query time)

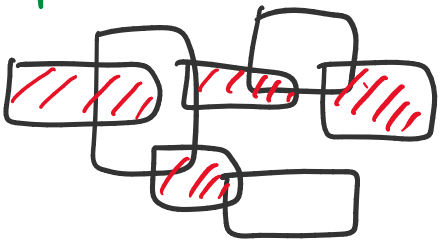
approximate ?

geometric approx alg'ms :

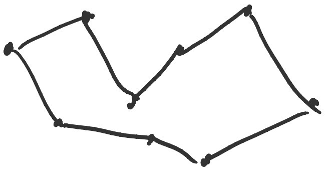
geom. set cover



geom. indep set



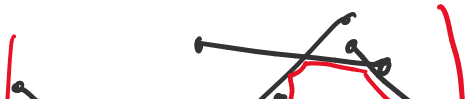
geom. TSP

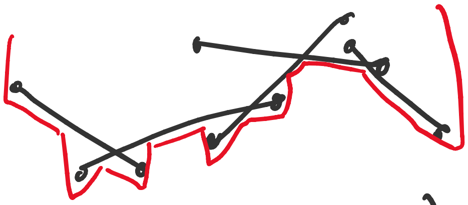


⋮

combinatorial geometry

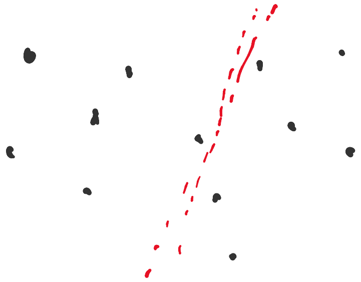
lower envelope of line segments





$\Theta(n\,\alpha(n))$

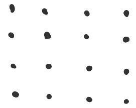
halving lines (k-set)



$O(n^{4/3})$

open

unit distance



$O(n^{4/3})$

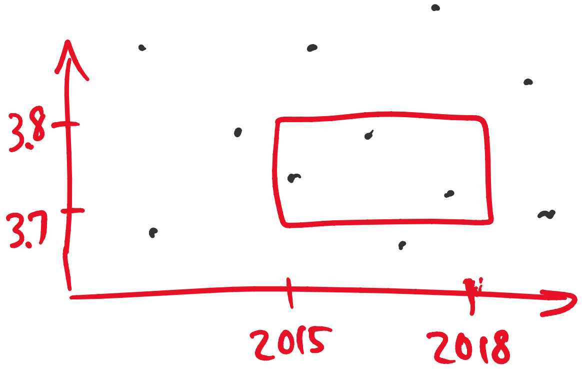
open

Orthogonal Range Searching

Store n pts in $\mathbb{R}^d$ in data structure s.t.

given query range q, find pts in q

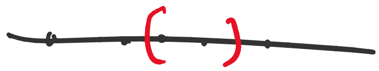
axis-aligned rectangle/box



diff versions:

- report all,
- decide emptiness
- count ←
- sum of weights
- max weight
- ⋮

(median weight,
most freq. weight
report all colors)
...

1D: 

space $O(n)$

preproc time $O(n \log n)$ by sorting

query time $O(\log n)$ for counting/emptiness

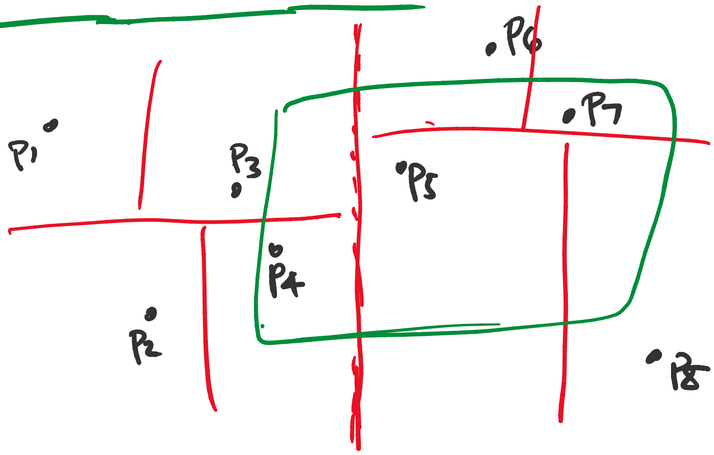
$O(\log n + k)$ for reporting

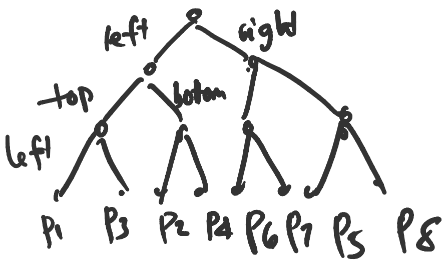
output size

(optimial in comparison model)

2D?

Method 1: "k-d Tree"





each node → rectangular cell

space $\boxed{O(n)}$

preproc. $P(n) = 2P(\frac{n}{2}) + O(n)$

median-finding

$\Rightarrow \boxed{O(n \log n)}$

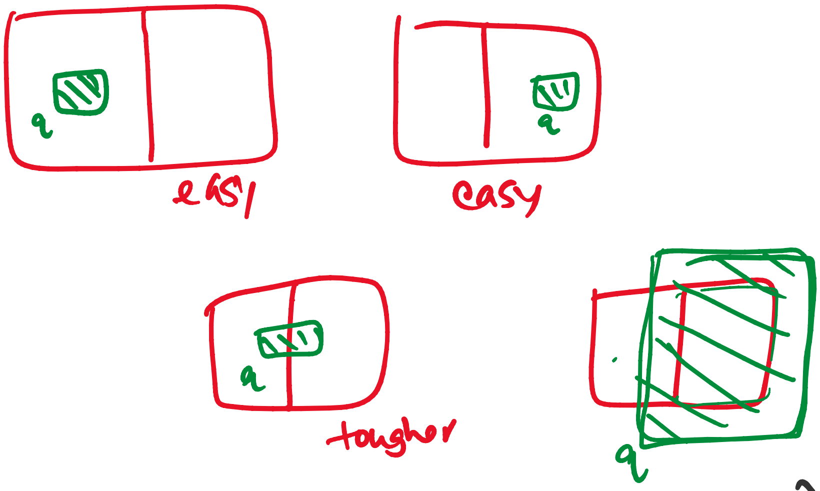
query alg'm, given rectangle q: //counting

if q does not intersect node's cell) return 0

else if q completely contains node's cell

return # pts in cell

```
if q does not ...
else if q completely contains node's cell
        return # pts in cell
else    recurse on both children
        return sum
```

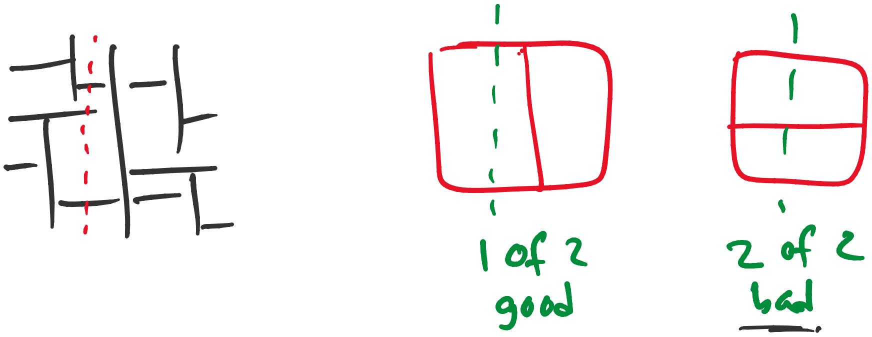


Analysis: query time = O(# cells visited)

= O(# cells crossing ∂q)

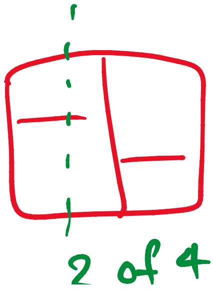
boundary has 4 line segs

How many cells in k-d tree can a vertical line cross?



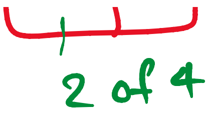
$f(n) = 2f(n/2) + 1$

$\Rightarrow$ O(n) bad!



$f(n) = 2f(n/4) + 1$

$\Rightarrow O(n^{\log_4 2})$



$\Rightarrow O(n \quad)$

$= O(\sqrt{n})$

$\Rightarrow$ query time $O(4 \cdot f(n))$

$= \boxed{O(\sqrt{n})}$

(+k for reporting)

Higher-d:

$$f(n) = 2^{d-1} f\left(\frac{n}{2^d}\right) + 1$$

$$\Rightarrow O\left(n^{\frac{d-1}{d}}\right) = \boxed{O\left(n^{1-1/d}\right)}$$