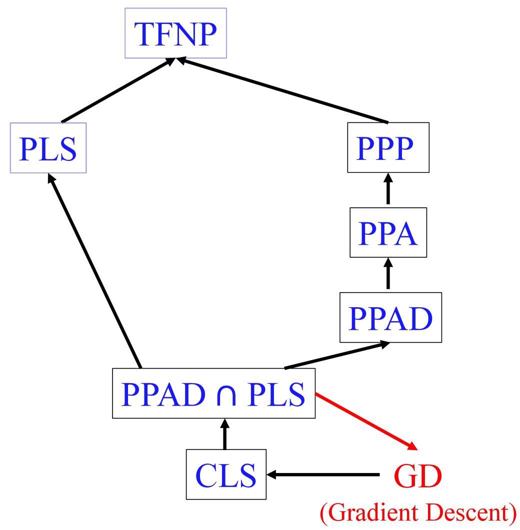
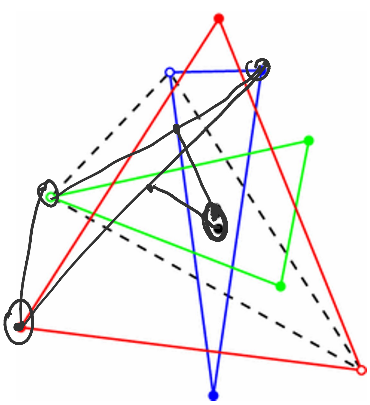
Open Problem Session I (CLS)

Friday, January 27, 2023 9:39 AM



1. Colorful Caratheodory Theorem (CCT)



$(d+1)^{(d+1)}$ panchromatic simplicies.

1. Is this CLS-hard? OR is there a sub-exponential time algo?

2. For $d \geq 2$ we know there exists more than 1 soln. what is the complexity of finding say 2 solns? OR what's the complexity of finding the # of solns?

2. Contraction Map (Banach Fixed Point)

Given: $f: [0,1]^n \to [0,1]^n$. given by an arithmetic circuit.

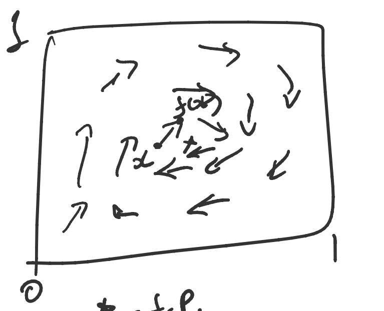
$\lambda, \varepsilon < 1$. p-norm

$-^n$. $\|f(x) - f(y)\|_p > \lambda \|x-y\|_p$

Given: $\lambda, \varepsilon < 1$. p-norm

Goal: either find $x, y \in [0,1]^n$ s.t. $\|f(x) - f(y)\|_p > \lambda \|x-y\|_p$

OR find $x \in [0,1]^n$ s.t. $\|f(x) - x\|_p \le \varepsilon \left(\sim \frac{1}{2^n}\right)$



$x^* = $ f.p.

$\|f(x) - x^*\|_\infty \le \lambda \|x - x^*\|_\infty$

We know:

1. Unique fixed-point
2. For $\lambda = (1-\delta)$, if $\delta \le \frac{1}{\mathrm{poly}(n)}$ then easy (say $p=\infty$)

$$\|f^2(x) - f(x)\|_p \le \lambda \|f(x) - x\|_p$$

$$\|f^3(x) - f^2(x)\|_p \le \lambda \|f^2(x) - f(x)\|_p$$

$$\le \lambda^2 \|f(x) - x\|_p \le \lambda^2$$

$$\|f^K(x) - f^{K-1}(x)\|_\infty \le \lambda^K \|f(x) - x\|_p \le \lambda^K \le \varepsilon$$

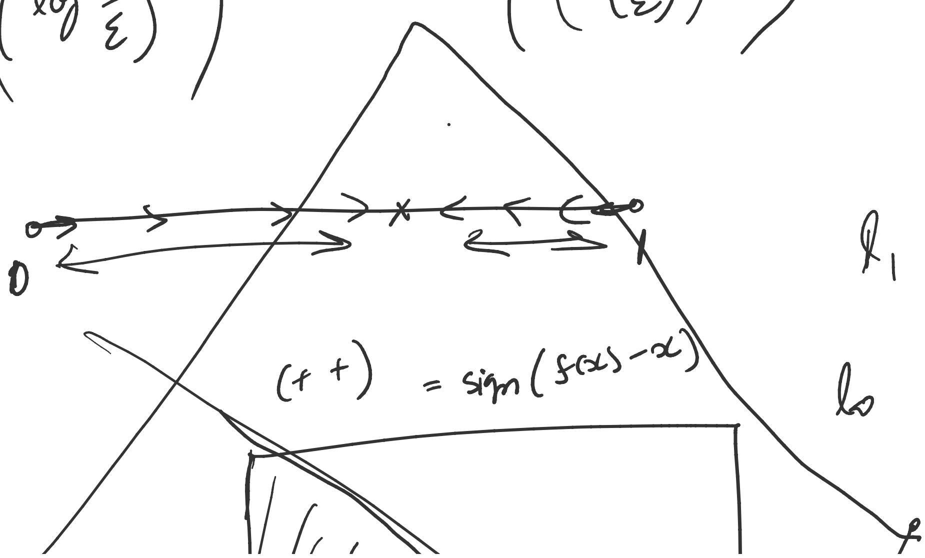
$$\Leftrightarrow (1-\delta)^K \le \varepsilon \Leftrightarrow \frac{1}{\varepsilon} \le (1-\delta)^K$$

$$\Leftrightarrow \lg \frac{1}{\varepsilon} \le K \lg(1-\delta) \le K\delta$$

$$\Leftrightarrow K \ge \left(\frac{1}{\delta}\left(\lg \frac{1}{\varepsilon}\right)\right)$$

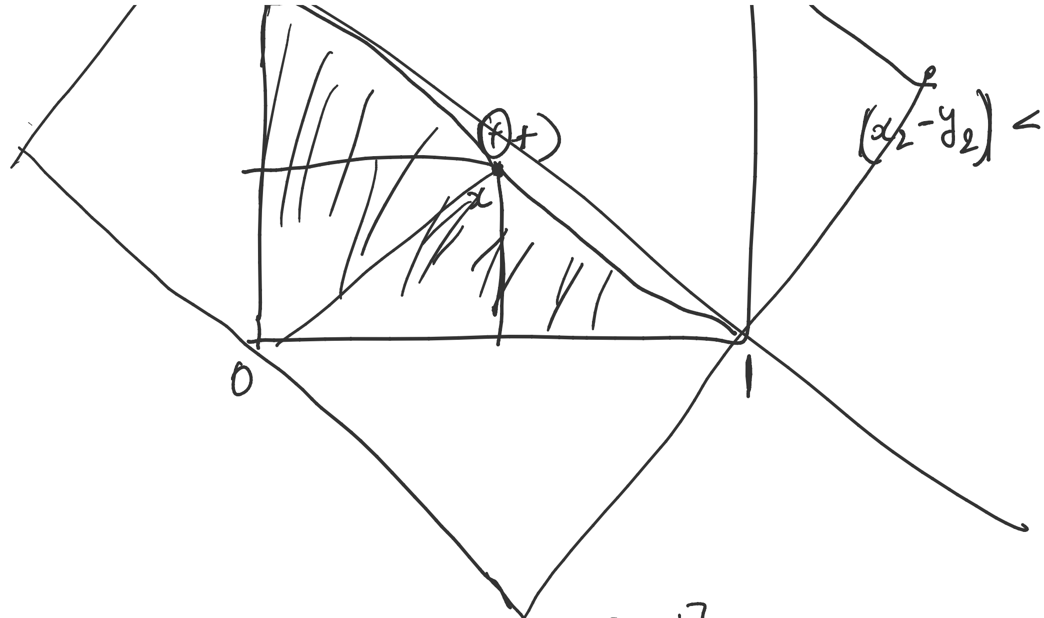
3. $p=2$ poly-time algo.

4. $O\left(\left(\log \frac{1}{\varepsilon}\right)^n\right) \rightarrow O\left(\left(\lg\left(\frac{1}{\varepsilon}\right)\right)^{n/2}\right)$



ℓ_1

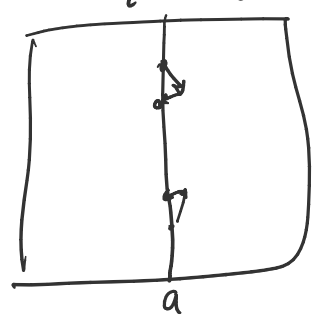
ℓ_∞ $O\left(\log \frac{1}{\varepsilon}\right)$



$a \in [0,1]$

$f : [0,1]^n \to [0,1]^n$

$g : [0,1]^{n-1} \to [0,1]^{n-1}$ $\qquad g(x) = f(a, x)\big|_{\{2 \dots n\}}$



$x, y \in [0,1]^{n-1}$

$$\|g(x) - g(y)\|_\infty = \|f(a,x)\big|_{\{2\dots n\}} - f(a,y)\big|_{\{2\dots n\}}\|_\infty$$
$$\le \|f(a,x) - f(a,y)\|_\infty$$
$$\le \lambda \|x - y\|_\infty$$

$f(x^*) = x^*$

$\|f(x) - x^*\|_p = \|f(x) - f(x^*)\|_p \le \lambda \underline{\|x - x^*\|_p}$

$g(x^*) = x^* \qquad x^* \in [0,1]^{n-1}$ $\qquad f(z^*) = z^*$

$f(a, x^*)\big|_{\{2 \dots n\}} = x^*$ If $f(a, x^*)_1 > a$ then

↓

1-D fixed point w/ oracle for solving $(n-1)$ dim problem.

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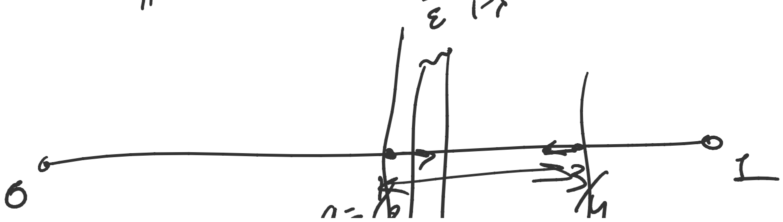
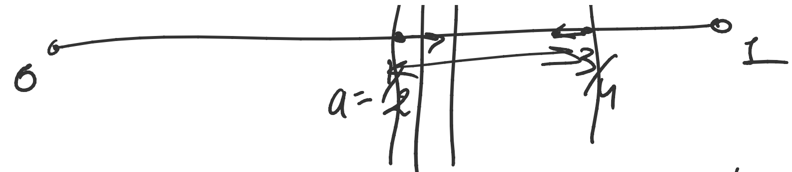
$\frac{1}{\varepsilon}$ $\frac{1}{1-\lambda}$





Lemma: $\{1, \dots, n\}$ $\quad D_1, D_2, \dots, D_K$ partition

$D_1 \to [0,1]^{D_1} \to P_1, \quad D_2 \to P_2, \quad \dots, \quad D_K \to P_K$

$P = P_1 \cdot P_2 \cdots P_K$

$1 \le i \le n/2 \qquad D_i = \{2i-1, 2i\} \to \log\left(\frac{1}{\varepsilon}\right)$

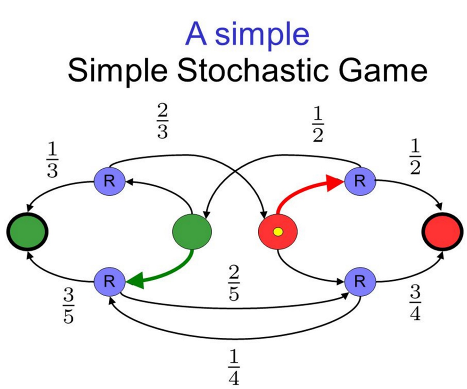
$P = \prod_{i=1}^{n/2} \text{time}(D_i) = \prod_{i=1}^{n/2} \log\left(\frac{1}{\varepsilon}\right) = \left(\log\frac{1}{\varepsilon}\right)^{n/2}$

Q: 1. Sub-exp? $\left(\log\frac{1}{\varepsilon}\right)^{n^{1-\delta}}$ for $\delta > 0$, $\left(\log\frac{1}{\varepsilon}\right)^{\frac{n}{\log n}}$?

2. Query lowerbound: we know if $n = 3$ then req. $\left(\log\frac{1}{\varepsilon}\right)^2$ queries.

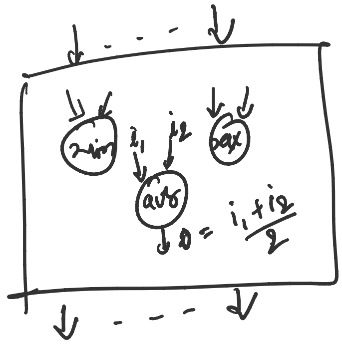
$\left(\log\frac{1}{\varepsilon}\right)^{\frac{n}{\log n}}$? $\qquad \left(\log\frac{1}{\varepsilon}\right)^{\lfloor n/2 \rfloor}$?

3. Simple Stochastic Games



$\exists$ Randomized sub-exp. time algo.

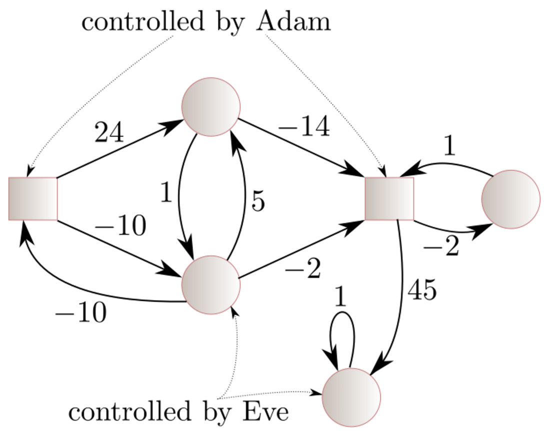
f :



4. Mean Payoff Games & Parity Games

quasi-poly (STOC'17 Best Paper)

MPG (randomized sub-exp.)



Parity

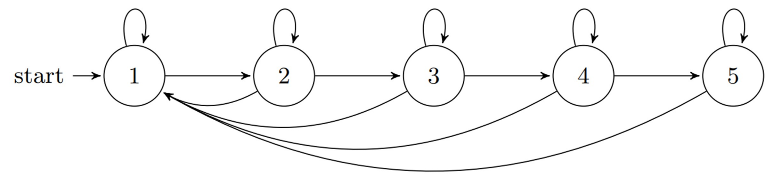


Figure 1.