Combinatorial Auctions (Spectrum Auction)

* Spectrum (Frequencies to broadcast)
  Every item here is (freq range, geographic location) for heterogeneous Items.

* Landing slots on airports

* Combinatorial Auction:
  n: agents (N) every agent i has value \( V_i(s) \) for any subset \( s \subseteq M \).
  m: different items (M)

Issues:

\[ \rightarrow \text{Representation of each } V_i \text{ req. } 2^n \text{ any numbers.} \]

\[ \rightarrow \text{Substitutes vs complements.} \]

\[ A, B \subseteq M \]

\[ V_i(A \cup B) \leq V_i(A) + V_i(B) \]

\[ V_i(A \cup B) \geq V_i(A) + V_i(B) \]

Easy case to analyze

Hard.

* Sell every item separately through "Single Item Auction" (SIA).

\[ V_i, \ldots, V_{im} \]

Mistake 1: Sequential SIA.

2-item similar

3-bidder

\[
\begin{align*}
\text{profit} & > \text{bidding} \\
\text{own} & \geq \text{bidding} \\
1000 & > \text{bidding} \\
\text{profit} & > \text{bidding}
\end{align*}
\]

If the first bidder skips the first auction, (or under bids)

Then she can win the second at $300.
Multi-Round Auction (SMR)

- Each item sold through an English auction
- Happens in rounds, starting with very low (reserve) price
- Each round has a fixed time after which agents can place bid. They can see the winning bid from the previous round.
Pros:

- Removes coordination issues for the bidders.
- Need not know the valuations upfront.
- Work well:
  - Similar items go for similar price
  - No reselling, or reselling at similar price.
  - Price discovery: winner pays price at mid-auction as correlated
    with similar prices at the end.
  - Exceeds the projected revenue.

Cons:

- Demand reduction. (With both substitutes & complements)
  \[ V_1(A) = V_1(B) = 10 \]
  \[ V_1(AB) = 20 \]
  \[ V_2(A) = V_2(B) = V_2(A, B) = 8 \]

\[ VCG: \quad \text{give both } A, B \text{ to 1, s.w. = 20} \]

\[ \text{payoff}_1 = 8 \]

\[ \text{Rev} = 8 \]

They will get A, B.
SAA (truthful bidding): I will get A, B
pay 16  \[ \text{Rev} = 16 \]

I (non-truthful): 1 bids only on A  \[ \text{B} \rightarrow 1 \text{gets A pays little} \]
2 bids on 1 " B \[ 2 \text{gets B pays little} \]

\[ \text{Rev} = \epsilon \text{ (very small)} \].

Exposure Problem (compensates)

\[ V_i(A) = V_i(B) = 0 \]

\[ V_i(A, B) = 100 \]

\[ V_2(A) = V_2(B) = V_2(A, B) = 75 \]

(75, 75)

Either 1 will win \( \{A, B\} \), pay 150!

SAA (truthful bidding): on 1 drops out at prices \( \{50, 50\} \)

Then 2 wins both \( \{A, B\} \) & pays 100!

Net utility < 0 in both.


* Improvement in SAA to eliminate exposure problem.

Allow package (combinatorial) bidding.

1. First run SAA for single items.

Then a round of package bidding.

2. Allow package bidding for fixed packages.

\[ \text{ABCD} \quad \text{EFGH} \]

\[ \text{AB} \quad \text{EF} \]

3. Allowed fixed # packages with upper bound on sizes

at most 12 items.
Since 2014:

step-1 (to buy spectrum): Reverse auction to sell up spectrum.

Step-2: Repack the remaining channels in a fixed range.

Algorithms.

Step-3: Forward auction to sell the freed up spectrum.

Upfront fix the frequency window to be freed.

85 60

Want to free up 100

Fix quantity to be freed up.

Whatever remains should "fit" in range

say 30 MHz - 60 MHz.

Step 1:

N: set of bidders willing to sell

v_i: value of agent i to its channel

b_i: bid at which it is willing to sell.

r_i: reserve it owns.

(revelation)

Direct mechanism:

Free = N, To Pack = \emptyset

while Free \neq \emptyset s.t. To Pack \cup i \notin \emptyset can be packed in 30-60 MHz range

- move one such i from Free to ToPack.

\( \Rightarrow \)\n
"Will me to choose?"
With one to choose?
The least bid agent or
least bid/unit spectrum or
most desirable spectrum usage.

Repacking: packing problem + coloring (NP-hard problems)

Two TV channels in the
same geographic location
should not “overlap”.

Colors: frequency bands.