Mechanism Design: Auctions

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Designing rules "of the game" s.t. desired outcome is achieved.

★ Auctions
- seller/auctioneer
- sell single item (indivisible)
- set of agents/bidders.

v_i = value of bidder i for the item.

What if v_i's are known?

$$\text{Price} = \max_i v_i - \epsilon$$

Issue: v_i is a "private information" of agent i.

★ Sealed Bid Auction:

(1) Bidders bid: b_i is bid of agent i (b_i need not be v_i) in a sealed envelope

(2) Auctioneer looks at all the bids & decides the winner/allocation

winner = Highest bidder = $\arg\max_i b_i$

(3) & what the winner pays. P.

$$\text{utility}(i) = U_i = v_i - P \quad \text{if } i \text{ is a winner (quasi-linear utility)}$$
$$= 0 \quad \text{o.w.}$$

★ $P = 0$ (pay nothing) ✗

★ First-price: pay your bid. Key $P = b_{i^*}$

i^* is the winner

v_1	v_2	v_3
200	600	500
150	375	400
200	465	450

i^* is the ...

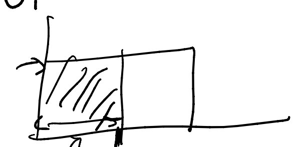
☆ Second-price : pay second highest bid

i^* pays $p = \max_{i \neq i^*} b_i$

200 $\quad \boxed{465} \quad 450 = p$

☆ First-Price :

2-bidders $\quad v_1, v_2 \sim U[0,1] \rightarrow$ Types θ_i (Private information)



Suppose, $\boxed{b_2 = \frac{v_2}{2}}$

If bidder 1 bids b_1

$$\max_{b_1} (v_1 - b_1) \Pr[b_2 \le b_1] \; \cancel{+ 0 \Pr[b_2 > b_1]}$$

$$= \max_{b_1} (v_1 - b_1) \Pr[v_2 \le 2b_1]$$

$$= \max_{b_1} (v_1 - b_1)(2b_1)$$

$$\frac{d}{db_1} (v_1 - b_1) 2b_1 = 0$$

$$v_1 - 2b_1 = 0 \Rightarrow \boxed{b_1 = \frac{v_1}{2}}$$

$\left(\frac{v_1}{2}, \frac{v_2}{2}\right)$ is a Bayesian NE

↓ Generalization

n-players. $\quad v_1, \ldots, v_n \sim U[0,1]$

Fix : $b_k = \frac{(n-1)v_k}{n} \quad \forall k \neq i$

Agent i wants to bid b_i s.t.

$$\arg\max_{b_i} (v_i - b_i) \Pr[b_k \le b_i, \forall k \neq i]$$

$$\arg\max_{b_i} (V_i - b_i) \cdots$$

$$= \frac{n-1}{n} V_i$$

Bidding $\frac{n-1}{n} V_i$ is Baeysian NE

→ what if arbitrary distribution?

→ " " diff arbitrary distributions?

very complex. Hard reason about.

✩ Second-price

winner $i^* = \arg\max_i b_i$ $\quad b_1 \geq b_2 \geq \cdots \geq b_n$ (winner: b_1, payment: b_2)

Payment $p = \max_{i \neq i^*} b_i$

[Vickrey '61] : (Bidding your true value is the best strategy.)

Bidding $b_i = V_i$ is dominant strategy

For every agent i, $b_i = V_i$ is … ($\equiv b_i = V_i$ is DSNE)

no matter what b_{-i} is.

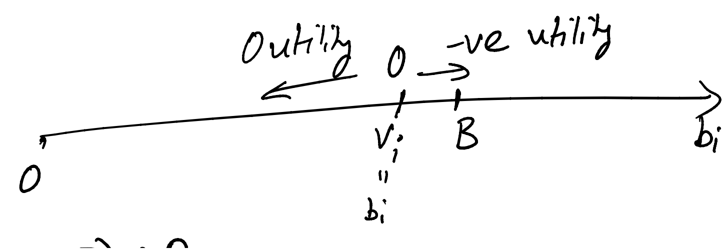
Pf: Fix an agent i. Two possible outcomes.

i wins $\Rightarrow U_i = V_i - \max_{k \neq i} b_k = B$

i looses $\Rightarrow U_i = 0 \quad (b_i \leq \max_{k \neq i} b_k = B)$

(Payment of the winner is independent of his bid)

Case I : $V_i \leq B \Rightarrow b_i = V_i$ is the best



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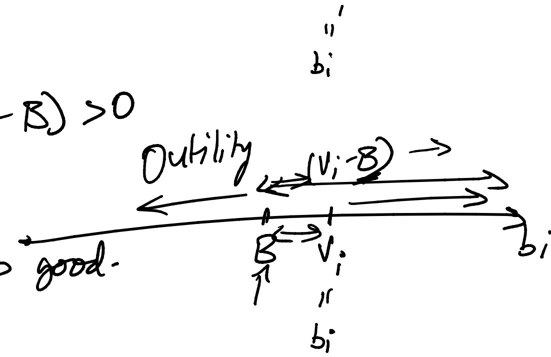
Case II: $V_i > B \Rightarrow (V_i - B) > 0$



$\Rightarrow b_i = V_i$ is as good.

Bidding your true value is dominant strategy

|||

Dominant strategy Incentive Compatible. (DSIC).

1. DSIC = truthful reporting.
2. Social welfare maximizing allocation.
3. polynomial-time computable (winner, payments)

Awesome Auctions.

Vickrey is an awesome auction.

English Auction (open-bid auctions): (EBay auction)

↳ Auctioneers announces a very low price 0.

There is a set of buyers willing to buy at the current price

↳ keep increasing price until one bidder remains.

$$\geq v_n \geq 0$$

$$v_1 \geq v_2 \geq \dots$$

Price = 0

Price = $v_n + \epsilon$

$v_1 = \cdot\cdot\cdot$

Price = 0 ↑

Price = $v_n + \epsilon$

⋮

Price = $v_2 + \epsilon$

second-price auction.

☆ Dutch Auction:

- Auctioneer starts with very high price
  - that no one wants to pay.
- Then keep decreasing until at least one agent is willing to pay.

$$v_1 \geq v_2 \geq \cdots \geq v_n$$

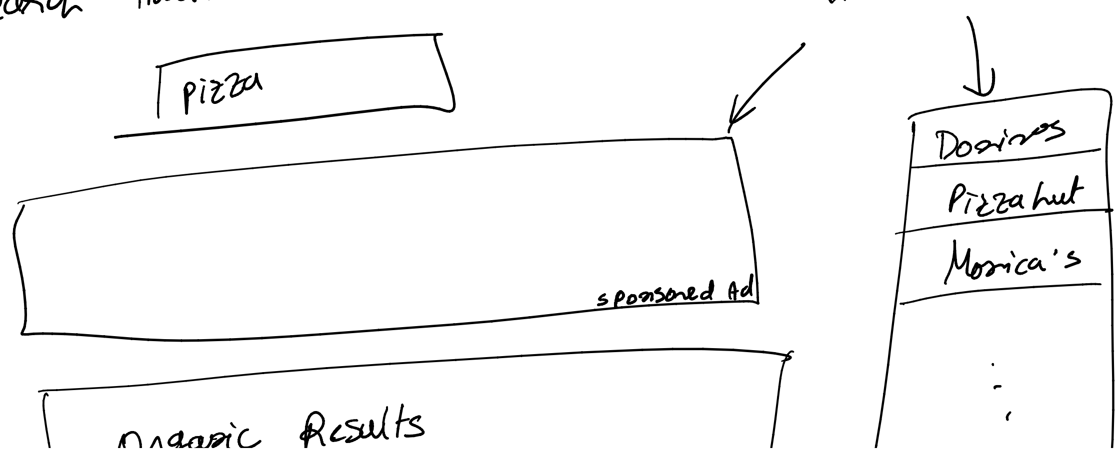
↓ Price = $P > v_1$

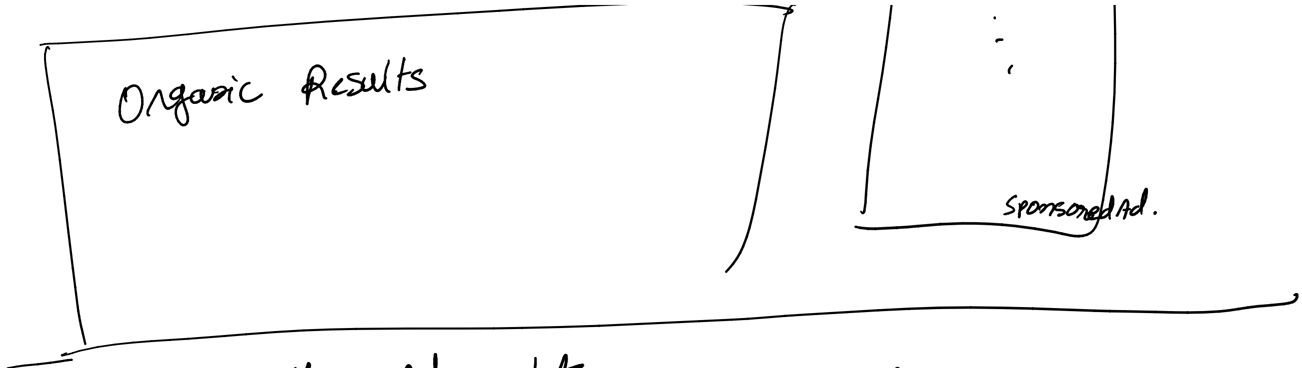
↑

$P = v_1$

close to first-price auction.

☆ Search Auction.

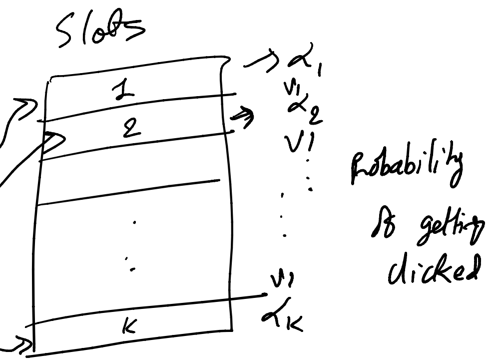




★ Auction to sell Ad slots.

Google (keyword auction)

① Find the set of advertisers whose product matches the query.



② $b_1 \geq b_2 \geq \dots b_K \geq b_n$

③ payment of bidder i is b_{i+1}

Q: Is this a truthful auction?

Is $b_i = v_i$ is DS ?

$i \leq K$

$$U_i = \alpha_i (v_i - b_{i+1})$$