Security Games

Tuesday, October 6, 2020 1:50 PM

Zero-sum Games: $(A, -A)$ — Alice = leader; maximizer → minimizer

For the leader

max min = Stackelberg.

$$\underset{x}{\text{argmax}} \min_{y} x^T A y = \text{leader's strategy in Stackelberg game.}$$

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n-targets to defend

$[n] = \{1, \ldots, n\}$

Defender
Leader

Set of sources to protect the targets.

↓

Set of possible defence strategies (pure)

$\mathcal{E} \subseteq \{0, 1\}^n$

$e \in \mathcal{E}$

$e_i = 1 \Rightarrow$ target i is defended as per e.

Follower
Attacker
can only attack one target
$i \in [n]$

... reward if i ...

as per e.

r_i : reward if i is defended ≥ 0

c_i : cost if i is not defended ≤ 0

s_i : reward if i is not defended ≥ 0

ξ_i : cost if i is defended ≤ 0

Pure play: (e, i)

$$\underline{r_i e_i + c_i (1 - e_i)} \qquad \underline{s_i (1 - e_i) + \xi_i e_i}$$

mixed-play: $p \in \Delta(\mathcal{E})$ $\qquad y \in \Delta([n])$

Defender's payoff: $\sum_{e \in \mathcal{E}} \sum_{i \in [n]} p_e \cdot y_i \left(r_i e_i + c_i (1 - e_i) \right)$

$$= \sum_{i \in [n]} y_i \left[r_i \Big(\underbrace{\sum_{e \in \mathcal{E}} p_e e_i}_{} \Big) + c_i \Big(1 - \Big(\sum_{e \in \mathcal{E}} p_e e_i \Big) \Big) \right]$$

marginal prob that i is defended $\to x_i$

$$= \sum_{i \in [n]} y_i \left(r_i x_i + c_i (1 - x_i) \right)$$

$$x \in P = \left\{ \sum_{e \in \mathcal{E}} p_e \cdot e \;\middle|\; p_e \in \Delta(\mathcal{E}) \right\}$$

$$\left\{ x \in [0,1]^n \;\middle|\; \sum^{n} x_i \leq k \right\}$$

$$= \left\{ x \in [0,1]^n \;\middle|\; \sum_{i=1}^{n} x_i \le k \right\}$$

$$= \sum_{i \in [n]} x_i \big(\underbrace{y_i (r_i - c_i)}_{w_i} \big) + \sum_{i \in [n]} c_i y_i$$

constant given y

Defender's Bestresponse (DBR): Given y,

$y \to w \qquad w_i = y_i (r_i - c_i) \ge 0$

$$\max_{e \in \mathcal{E}} \; e \cdot w$$

$\uparrow$ combinatorial set

combinatorial problem.

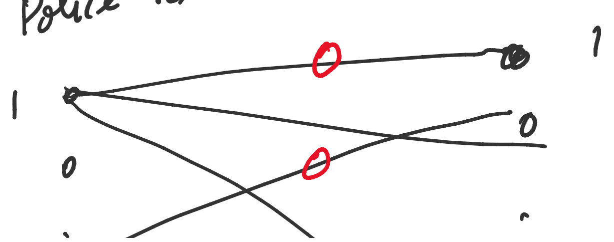
Examples ($\mathcal{E}$):

① ORD: n-gates k-police personals.

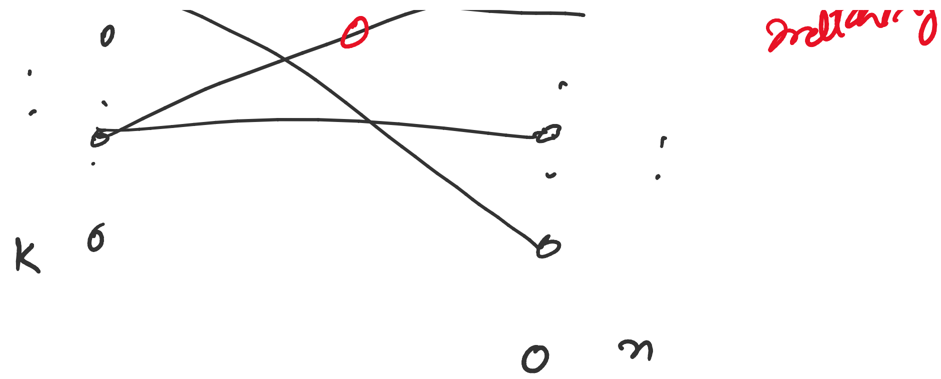
$k < n$

$$\mathcal{E} = \left\{ e \in \{0,1\}^n \;\middle|\; \sum_{i=1}^{n} e_i \le k \right\}$$

② Police Personals   Gates

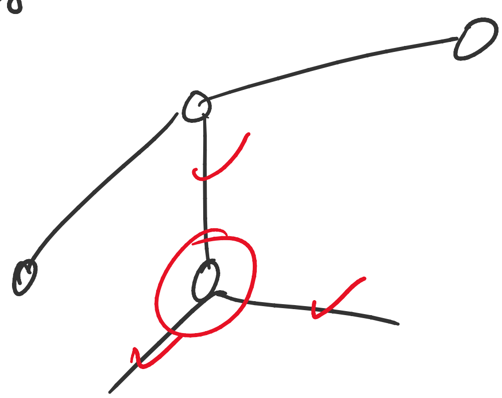


matching



$\mathcal{E}$ = All matchings

(3) k-patrols. Protect n/w
(eg. street n/w of a city)



$\mathcal{E}$ = set of all vertex covers

→ More General Problem:

Airmarshal Problem:

↳ protects flight A & B only if destination of A = source of B or vice-versa.

$S_i \subseteq [n]$ $\quad S_1 \ldots S_m$

$K \subseteq \{1 \ldots m\}$ s.t.

pick k sets

$S_i = \ldots$

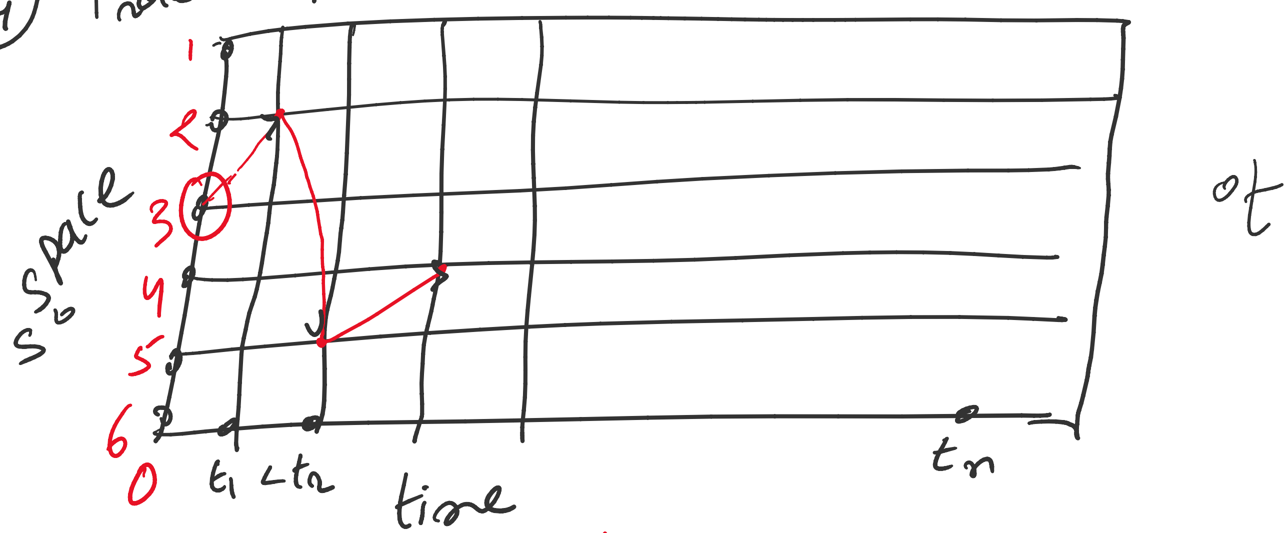
Coverage : pick k sets s.t.

$\bigcup_{i \in K} S_i$ is maximized.

e_i as a set of targets

$\mathcal{E} = \left\{ \bigcup_{i \in K} S_i \;\middle|\; |K| \le k \right\}$

④ Protect parts (Patrolling Forest)



⇓

$\mathcal{E}$ = select k s-t paths s.t.

max # targets are defended

for max # times