

Spring 2026, CS 583: Approximation Algorithms

Homework 1

Due: 02/12/2026 in Gradescope

- Each homework can be done in a group of size at most two. Only one homework needs to be submitted per group. However, we recommend that each of you think about the problems on your own first.
- Homework needs to be submitted in pdf format on Gradescope. See <https://courses.grainger.illinois.edu/cs374a11/fa2025/hw-pol.html> for more detailed instructions on Gradescope submissions.
- Follow academic integrity policies as laid out in student code. You can consult sources but cite all of them including discussions with other classmates and LLMs. Write in your own words. See the site mentioned in the preceding item for more detailed policies including specific instructions on using LLMs.
- Read through all the problems and think about them and how they relate to what we covered in the lectures. Solve as many problems as you can but I expect that you will submit 5.
- Please write clearly and concisely - clarity and brevity will be rewarded. Refer to known facts as necessary.

Problem 1. The Greedy algorithm for Max- k -Coverage and Set Cover can be implemented even if the sets in the set system are implicitly defined as long as we have the following oracle: given a subset $\mathcal{U}' \subseteq \mathcal{U}$ of the remaining uncovered elements from \mathcal{U} , return the set in the set system that covers the maximum number of elements from \mathcal{U}' . Some times we only have an *approximate* oracle. Suppose we only have an α -approximate oracle for some $\alpha \leq 1$ which outputs a set from the set system that has at least α times the number of elements covered by the best set.

- Show that with an α -approximate oracle Greedy gives a $(1 - e^{-\alpha})$ -approximation. Note that this bound is better than the easier bound of $\alpha(1 - 1/e)$.
- The multiple knapsack problem (MKP) is the following. Like in the standard knapsack problem the input consists of n items, each of which has a profit p_i and a size s_i . However, we are now given m knapsacks with capacities B_1, B_2, \dots, B_k and items can be packed in these knapsacks without violating their capacities. Consider a Greedy algorithm that picks each knapsack in turn and packs it using a α -approximate algorithm for the single knapsack problem over the remaining items. Show that if all knapsack capacities are the same then this yields a $(1 - e^{-\alpha})$ -approximation. What do you obtain via the FPTAS you have seen for Knapsack?
- **Extra Credit:** Assume $\alpha = 1$. Show that the Greedy algorithm gives a $1/2$ -approximation even if the capacities are non-uniform and the order of the knapsacks is chosen arbitrarily.

Problem 2. We saw a randomized rounding algorithm with alteration for Set Cover that converts a fractional solution x to the LP relaxation to a feasible solution with expected cost at most $(1 + \ln \Delta) \text{OPT}_{LP}$ where Δ is the maximum set size. We also saw a simple deterministic rounding algorithm that yields an f -approximation where f is the maximum frequency (recall that $f = 2$ for Vertex Cover). Can we obtain the best of these two algorithms? By adapting the alteration algorithm suitably show that one can obtain a solution with expected cost at most $(1 + (f - 1)(1 - e^{-\frac{\ln \Delta}{f-1}})) \cdot \text{OPT}_{LP}$. Prove that this bound is always at most $\beta = \min\{f, 1 + \ln \Delta\}$, and is in fact a constant factor smaller than f when $f = 1 + \ln \Delta$; this bound is better than choosing the better of the two previous algorithms.

Problem 3. Problem 1.4 from Williamson-Shmoys book.

Problem 4. Consider a budgeted version of the maximum coverage problem. We are given m sets S_1, S_2, \dots, S_m , each a subset of a set \mathcal{U} . Each set S_i has a non-negative cost c_i and we are also given a budget B . The goal is to pick sets of total cost at most B so as to maximize the number of elements covered.

- Show that if $c_i \leq \epsilon B$ for $1 \leq i \leq m$ then the Greedy algorithm yields a $1 - 1/e - \epsilon$ approximation (assume ϵ is sufficiently small if necessary).
- For any fixed $\epsilon > 0$ obtain a $1 - 1/e - \epsilon$ approximation. In fact you can obtain a $(1 - 1/e)$ -approximation.

Problem 5. Problem 13.4 from Vazirani book. Do any two of the three subquestions. Extra credit for all three.

Problem 6. Suppose we are given n points p_1, p_2, \dots, p_n in the Euclidean plane. We wish to cover the points with as few unit radius disks as possible. Note that the disks can be positioned anywhere in the plane.

- How would you implement the Greedy algorithm for this Set Cover instance in polynomial time?
- Describe a constant factor approximation for this covering problem. It may be easier to first think of the case where you want to cover with squares with unit side length.

Problem 7. Knapsack Cover is the following problem. We are given n items where each item i has a non-negative size s_i and a non-negative cost c_i , and a knapsack of capacity B . The goal is to choose a minimum cost subset of items such that their total size is at least B . Obtain an FPTAS for this problem. You may first want to think of a constant factor approximation.

Optional Problem. Problem 3.6 from Williamson-Shmoys book.

Optional Problem. The multiple knapsack problem (MKP) is the following. Like in the standard knapsack problem the input consists of n items, each of which has a profit p_i and a size s_i . However, we are now given m knapsacks with capacities B_1, B_2, \dots, B_k .

- Describe a pseudo-polynomial time exact algorithm for the problem when $k = 2$.
- Prove that even for $k = 2$ and unit profits the problem is NP-Hard. Also prove that there is no FPTAS for the same setting.