

★ Atomic Routing Games:

- $G = (V, E)$ directed n/w.
- $N = \{1, \dots, n\}$ agents / players.
- $i \in N$, wants to go from s_i to t_i
 $s_i, t_i \in V$

P_i : set of $s_i \rightarrow t_i$ paths.

$P_i \in P_i$: ^{pure} strategy of player i

$P = (P_1, \dots, P_m)$

$f_e^P = f_e = \#$ agents taking edge e in P .

$$C_i(P) = \sum_{e \in P_i} c_e(f_e)$$

$\forall e \in E$:

$c_e: \{0, 1, \dots, n\} \rightarrow \mathbb{R}_+$ non-dec non-seg.

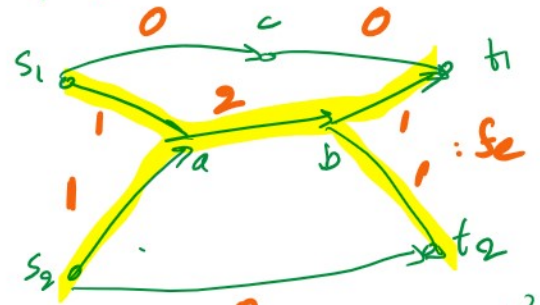
$$\text{Cost}(P) = \sum_{i \in N} C_i(P)$$

$$= \sum_{i \in N} \sum_{e \in P_i} c_e(f_e)$$

$$= \sum_{e \in E} c_e(f_e) \left(\sum_{\substack{i \in N: \\ e \in P_i}} 1 \right) = \sum_{e \in E} f_e \cdot c_e(f_e)$$

$$\text{Cost}(P) = \sum_{e \in E} f_e \cdot c_e(f_e)$$

$$P_1 = \{s_1 - c - t_1, s_1 - a - b - t_1\}$$



$$P_2 = \{s_2 - a - b - t_2, s_2 - t_2\}$$

P is NE iff $\forall i: C_i(P_i, P_{-i}) \leq C_i(q_i, P_{-i}) : \forall q_i \in P_i$

> atomic / n. m.

2.4
(non-trivial)

$\langle 4, 1, 1, 1, \dots \rangle$
 i

$$= \sum_{e \in E} c_e(s_e) \sum_{\substack{i \in N: \\ e \in P_i^*}} 1 = f_e^*$$

($\because c_e(s_e) = a_e + b_e$)

$$= \sum_{e \in E} f_e^* (a_e(s_e+1) + b_e)$$

$$= \sum_{e \in E} a_e \underbrace{f_e^* (s_e+1)}_{\substack{x \\ y}} + \sum_{e \in E} b_e f_e^*$$

$$\left(\begin{array}{l} x, y \in \{0, 1, 2, \dots\} \\ x(y+1) \leq \frac{5}{3}x^2 + \frac{1}{3}y^2 \end{array} \right) \text{ Magic!}$$

$$\leq \sum_{e \in E} a_e \left(\frac{5}{3} f_e^{*2} + \frac{1}{3} f_e^2 \right) + \frac{5}{3} \sum_{e \in E} b_e f_e^*$$

$$+ \frac{1}{3} \sum_{e \in E} b_e f_e$$

$$= \frac{5}{3} \sum_{e \in E} (a_e f_e^{*2} + b_e f_e^*) + \frac{1}{3} \sum_{e \in E} (a_e f_e^2 + b_e f_e)$$

$$= \frac{5}{3} \sum_{e \in E} f_e^* (a_e f_e^* + b_e) + \frac{1}{3} \sum_{e \in E} f_e (a_e f_e + b_e)$$

$$= \frac{5}{3} \sum_e f_e^* c_e(f_e^*) + \frac{1}{3} \sum_e f_e c_e(f_e)$$

$\because c_e(s_e) = 1$

$$\begin{aligned} & \left(\because \text{cost}(P) = \frac{1}{3} \sum_{i \in N} \text{cost}(P_i) \right) \\ & = \frac{5}{3} \text{cost}(P^*) + \frac{1}{3} \text{cost}(P) \end{aligned}$$

$$\sum_{i \in N} G_i(P_i^*, P_i) \leq \frac{5}{3} \text{cost}(P^*) + \frac{1}{3} \text{cost}(P) \quad \rightarrow (2)$$

Step-3
trivial

$$(1), (2) \Rightarrow \text{cost}(P) \stackrel{(1)}{\leq} \sum_{i \in N} G_i(P_i^*, P_i) \stackrel{(2)}{\leq} \frac{5}{3} \text{cost}(P^*) + \frac{1}{3} \text{cost}(P)$$

$$\Rightarrow \text{PoA} = \frac{\text{cost}(P)}{\text{cost}(P^*)} \leq \frac{\frac{5}{3}}{\left(1 - \frac{1}{3}\right)} = \frac{5}{2}$$

★ (x, u) - Smooth Game if $P: NE$
 $P^* \in \times_i P_i$

$$\sum_i G_i(P_i^*, P_i) \leq \lambda \sum_i G_i(P^*) + \mu \sum_i G_i(P)$$

(step-2)