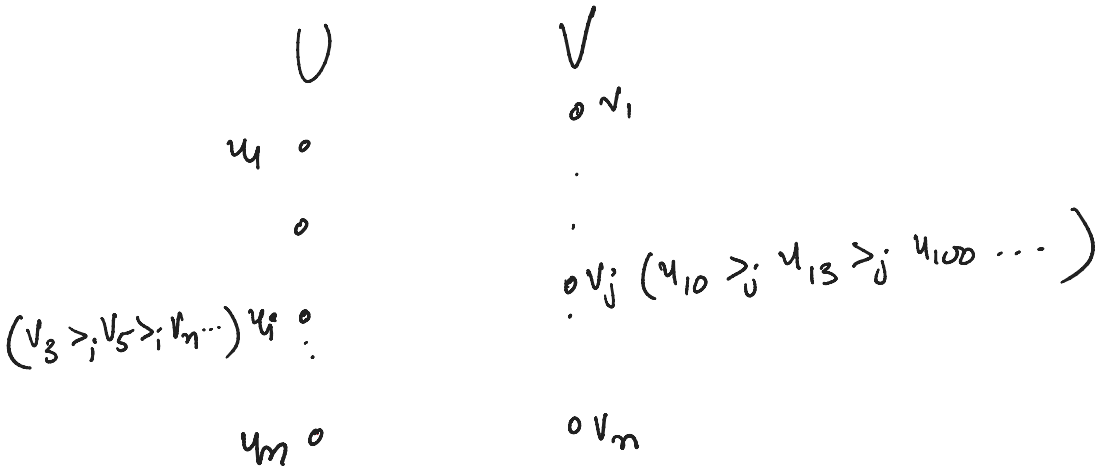
Memchanism Design without Money: Stable Matching, TTC

Thursday, September 7, 2023 10:56 AM

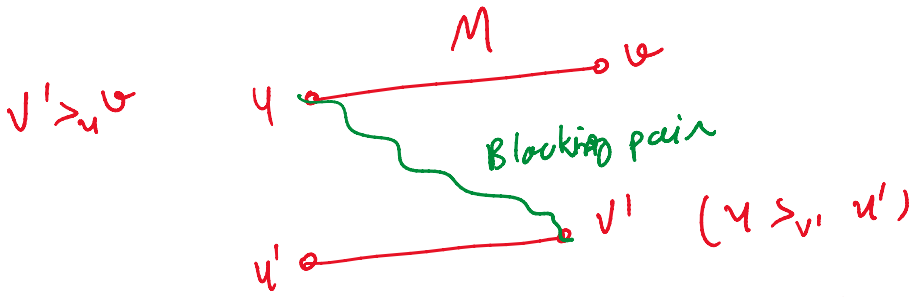
✡ Stable Matching (Marriage):

Applns: Hospital-resident matching
school / course seats assignment...



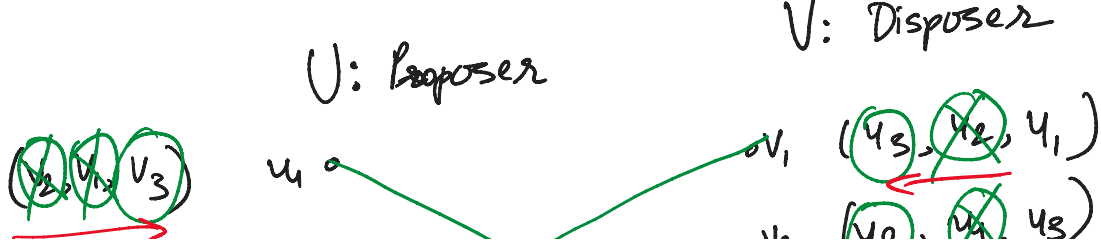
Goal: Find a "stable matching".

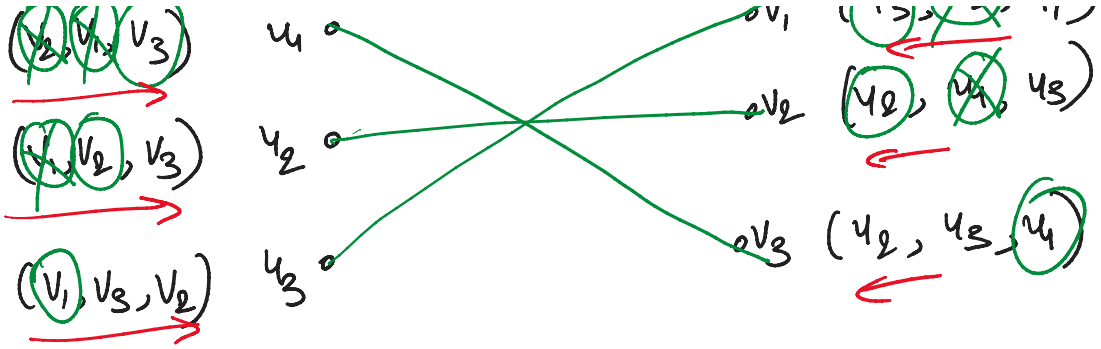
Blocking Pair:



We say that M is stable if there are no blocking pairs.

✡ Gale-Shapley (GS) algo: Differed Acceptance Algo.





Algo: In every round an unmatched vertex on U-side proposes to their next most preffered Vertex on V-side

OBS1: Has to terminate because in each round one of the u_i's goes down in their list.

Running Time: $O(n^2)$

OBS2: Algo o/p Perfect Matching.

Thm: GS Algo o/p a Stable Perfect Matching.

Pf: M: o/p of GS Algo. suppose M is not stable.

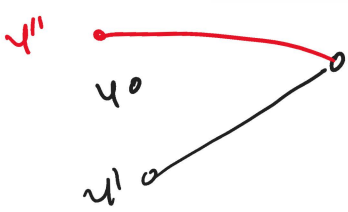
$\exists (u,v) \notin M$ & is a blocking pair.

Case I: u never proposed to v during the algo.

$\Rightarrow v'' > v \Rightarrow (u,v)$ is not blocking.

Case II: u did propose to v, but v rejected u for some other u'. $\Rightarrow u' >_v u$

Case II: u [illegible] for some other u'. $\Rightarrow u' >_v u$



$u'' >_v u' >_v u$ $\Rightarrow (u,v)$ is not a blocking pair

∎

Fun Fact:

$u \in U$: $opt(u)$ = Most preferred v s.t. $\{v \mid (u,v)$ is in some stable matching$\}$ by u.

$v \in V$: $opt(v)$ = Most preferred u s.t. $\{u \mid (u,v)$ in some stable matching$\}$ by v.

U-opt: M^u: $\{(u, opt(u)) \mid u \in U\}$ why matching? " "

V-opt: M^v: $\{(opt(v), v) \mid v \in V\}$

GS Algo o/ps M^u.

Message: Be the proposer.