Multi Parameter Auction: VCG

CS580

Ruta Mehta

Awesome Auction

(1) Truthful (DSIC)

(2) Social welfare maximizing

(3) Easy to compute

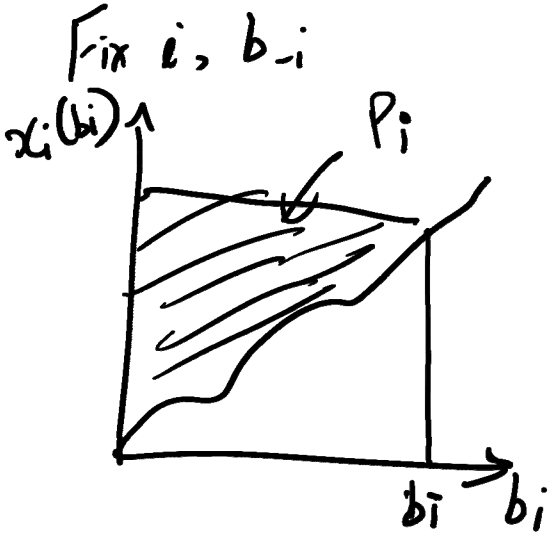
- poly-time

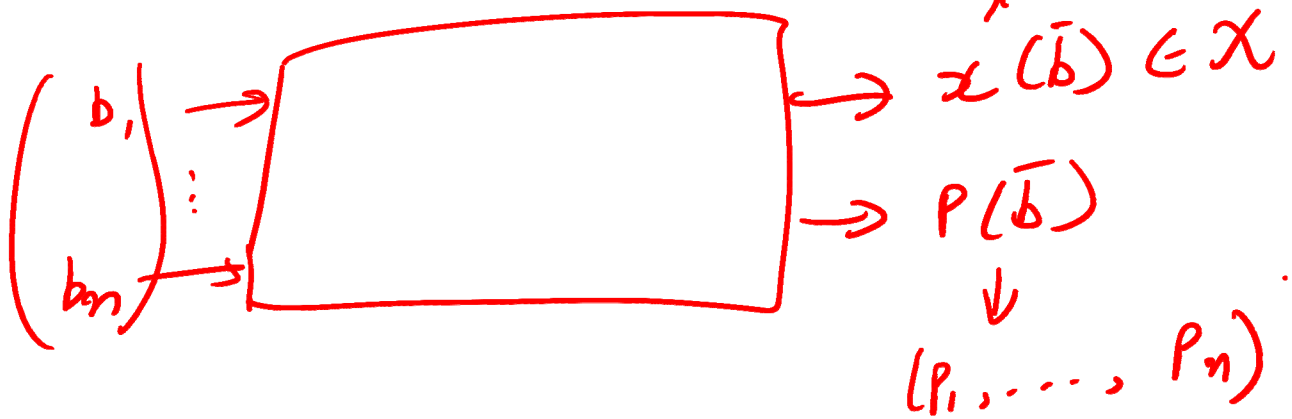
Recall: Single Parameter

$N = \{1, \ldots, n\}$ bidder , single stuff on auction

$i \in N$: Private value v_i / unit

bids b_i / unit





$u_i(\bar{b}) = v_i * x_i(\bar{b}) - p_i(\bar{b})$

DSIC $\iff$ $x(\cdot)$ is monotone.

Single Parameter: TV Ad. Example

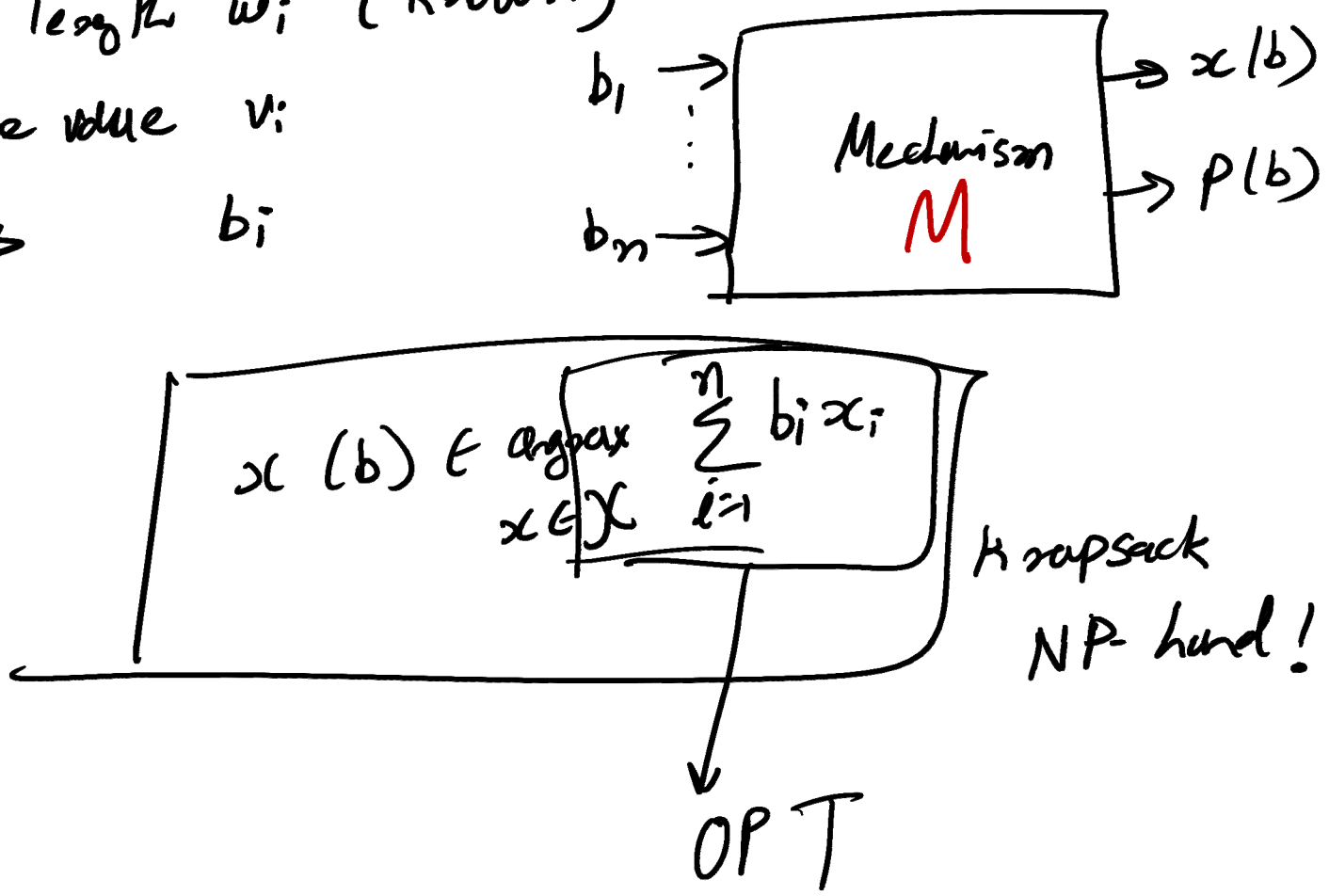
W: Ad window.

$X = \{\bar{x} \in \{0,1\}^n \mid \sum_{i=1}^{n} w_i \cdot x_i \leq W\}$

$i \in N$: Ad i of length w_i (known)

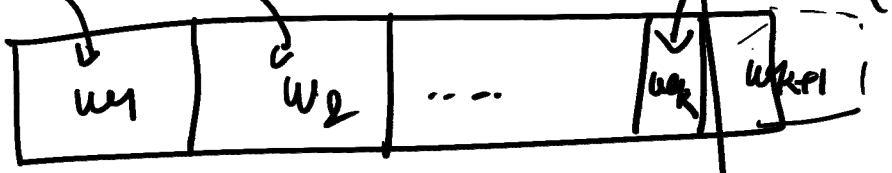
Private value v_i

Bids b_i



Single Parameter: TV Ad. Example

Greedy: $\frac{b_1}{w_1} \geq \frac{b_2}{w_2} \geq \cdots \frac{b_k}{w_k} \geq \frac{b_n}{w_n}$



Choice 1: Allocate $1 \to k$.

Choice 2: $i^* = \arg\max_i b_i$

Greedy x: If $\sum_{i=1}^{k} b_i > b_{i^*}$ then pick choice 1.

Else ($b_{i^*} > \sum_{i=1}^{k} b_i$) then allocate only i^*.

Claim: $\sum_{i=1}^{k} b_i x_i^{Greedy} \geq \frac{OPT}{2}$

MONOTONE.

$\Downarrow$

$\exists p$ s.t. (x^{Greedy}, p) is DSIC.

Single Parameter: TV Ad. Example

PTAS.: Find $(1-\varepsilon)$ - approx in poly-time. (achieve $[(1-\varepsilon)\,OPT]$ welfare)

↘ can be made monotone!

↓

Non-monotone Allocation!

[Chawla, Hartline, Immorlica '17]

Any allocation rule $x(\cdot)$ w/ downward closed X.

↳ if $x \in X$, $\forall y \le x$: $y \in X$

then $x(\cdot)$ can be made monotone.

Revelation Principle

Thm: If $\exists M$ that has Dominant (pure) Strategy Eq. (DSE)

Strategy bidder i: $s_i : V_i \to R_+$ $\left(s_i(V_i) \neq V_i\right)$

$(s_1, \ldots, s_n)$ are at DSE.

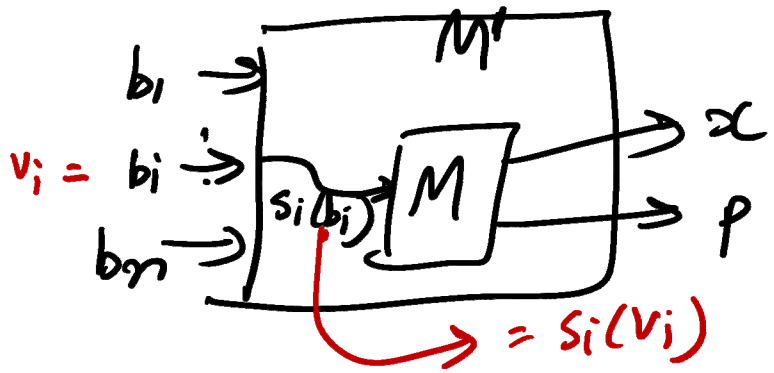
Then $\exists M'$ that is DSIC.

$\Downarrow$

$\exists$ DSE $s_i' : V_i \to R$

where $v_i \in V_i$ $s_i'(V_i) = V_i$

Pf:



General Case: Multi Parameter Setting

N: bidders. $M = \{$laptop, painting, pizza, pen, ... $\}$

Ω = all possible outcomes.

$i \in N$: $V_i : \Omega \to \mathbb{R}$ (private)

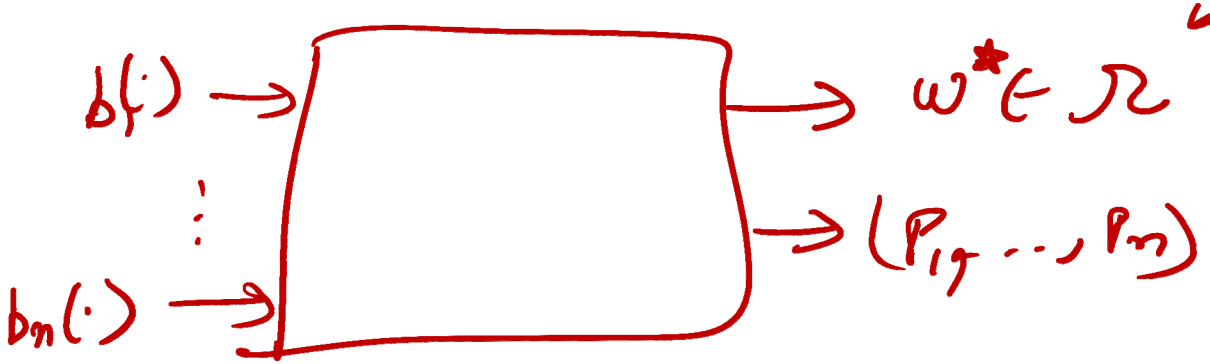
$b_i : \Omega \to \mathbb{R}$ (bid).

Single item

$\Omega = \{ i\text{-wins} \mid i \in N \}$

$V_i(k\text{-wins}) = V_i$ if $k = i$

$= 0$ o.w.



← s.w. maximizing.

Multi Parameter: VCG

S.W. maximization.

[Vickrey - Clark - Groves]

$$\omega^* \in \arg\max_{\omega \in \Omega} \sum_{i=1}^{n} b_i(\omega)$$

You Pick ω^*

"

agent i

$$p_i = \max_{\omega \in \Omega} \sum_{k \neq i} b_k(\omega) - \sum_{k \neq i} b_k(\omega^*)$$

$h_i(b_{-i})$

↳ Harm caused by i to all other agent's welfare.

$$U_i(\omega^*) = V_i(\omega^*) + \sum_{k \neq i} b_k(\omega^*) - h_i(b_{-i})$$

fixed no matter b_i

Multi Parameter: VCG

Thought exp:

① If i were to pick $w^* \in \arg\max_{w \in \Omega} V_i(w) + \sum_{k \neq i} b_k(w) - h_i(b_{-i})$ (constant)

$= \arg\max_{w \in \Omega} V_i(w) + \sum_{k \neq i} b_k(w)$

Auctioneer is picking $w^* \in \arg\max_{w \in \Omega} b_i(w) + \sum_{k \neq i} b_k(w)$.

Obs: Auctioneer is forced to do agent i's optimization (①)

iff $b_i = V_i$!!!

THEN: $b_i = V_i$ is Dominant strategy of agent i

$\equiv$ VCG is DSIC (Truthful)

Multi Parameter: VCG

Eg: Single item

$$\Omega = \{ i\text{-wins} \mid i \in N \}$$

(Private) $v_i(k\text{-wins}) = v_i$ if $k = i$

$= 0$ o.w.

Second price Auction!

Bid: $b_i(k\text{-wins}) = b_i$ o.w.

$= 0$

$$\omega^* \in \underset{\omega \in \Omega}{\operatorname{argmax}} \sum_{j=1}^{K} b_j(\omega) = \underset{i\text{-wins} \in \Omega}{\operatorname{argmax}} \; b_i$$

$$p_i = \max_{\omega \in \Omega} \sum_{k \neq i} b_k(\omega) - \boxed{\sum_{k \neq i} b_k(\omega^*)} = 0$$

Multi Parameter: VCG

(e x e)

k- identical items

keyword (Google) Auction

⋮