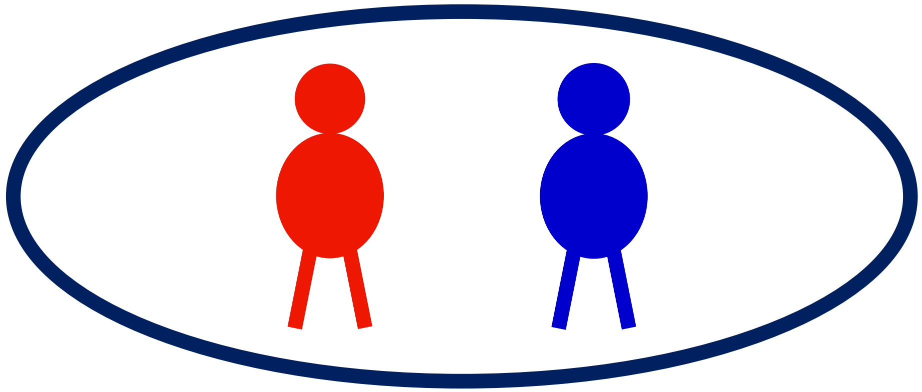
CS 580

Algorithmic Game **Theory**

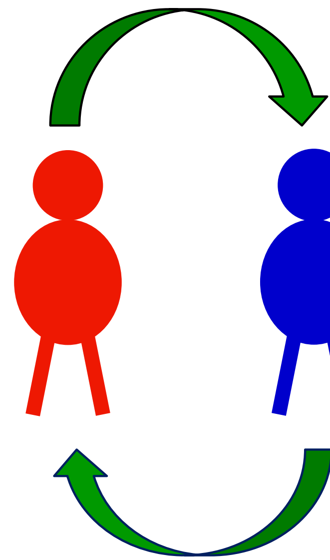
Instructor: Ruta Mehta

Game Theory

Multiple **self-interested** agents interacting in the same environment

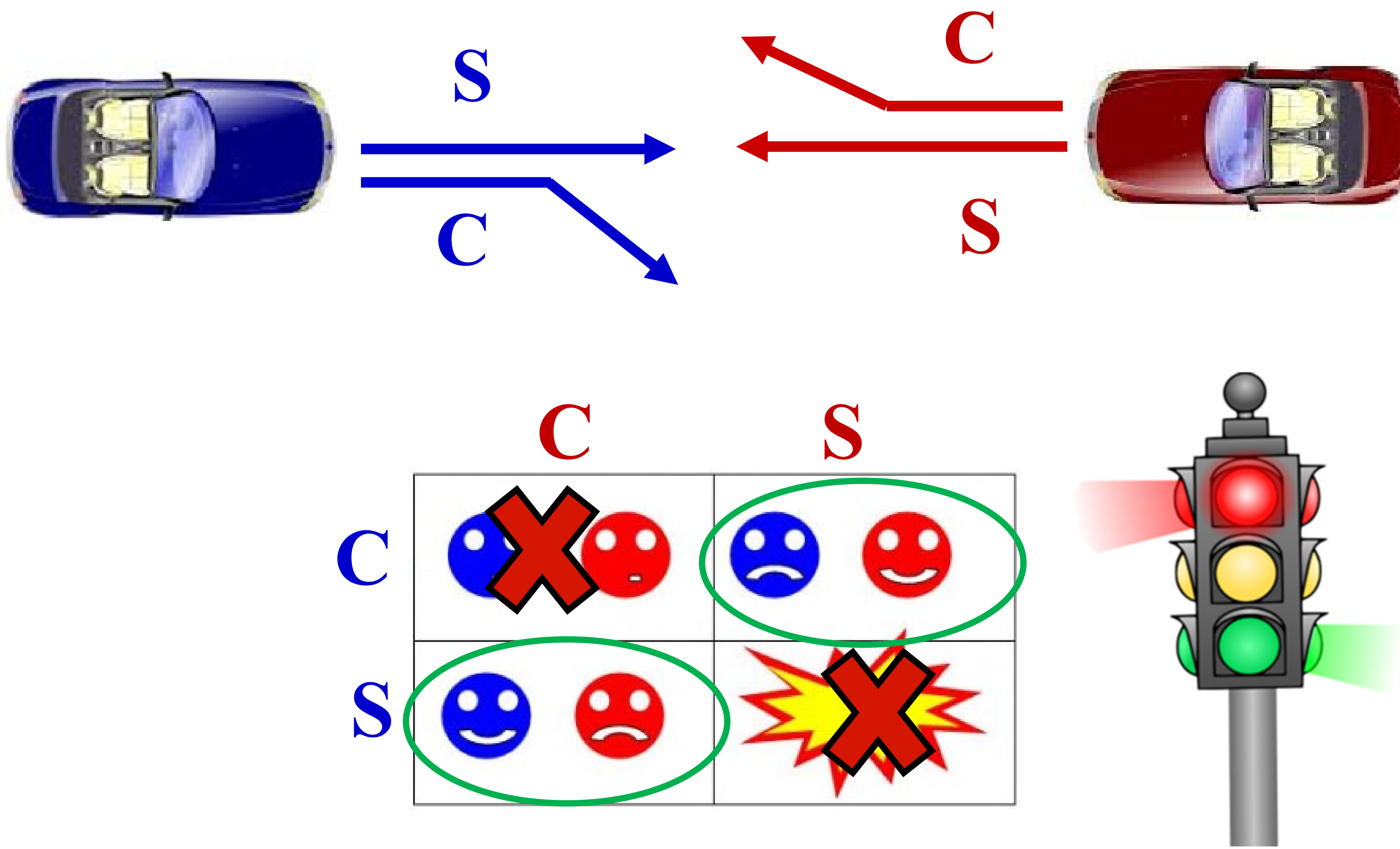


Deciding what **to do**.



Q: What to expect? How good is it? Can it be controlled?

Game of Chicken (Traffic Light)



Algorithmic Game Theory

AGT, in addition, focuses on designing efficient algorithms to compute solutions that are crucial (e.g., to make accurate prediction).

- **What to expect**

  Research-oriented Course

  - Exposure to key concepts and proof techniques from AGT
  - Explore research problems and novel questions

- **What is expected from you**
  - Pre-req: Basic knowledge of linear-algebra, linear programming, probability, algorithms.
  - Energetic participation in class
  - Research/Survey Project (individually or in a group of two).

- Instructor: Ruta Mehta (Me)
- TA: Rucha Kulkarni
- Office hours:
  - Ruta: Tue 2-3pm in Siebel 3218
  - Rucha: Thu 10-11am via zoom

Useful links

- Webpage: https://courses.engr.illinois.edu/cs580/fa2023
- Piazza Page: piazza.com/illinois/fall2023/cs580
- Slack: CS-580-Fall-2023
- Gradescope for grading

Check webpage/piazza at least twice a week for the updates.

HW0 is up.

- Grading:
  - 3 homeworks – 30% (10,10,10)
  - Research/Survey Project – 45%
    - Work – 20%
    - Presentation – 12.5%
    - Report – 12.5%
  - Final Exam or HW4 – 22%
  - Class participation – 3%

HW0 is for self-study (not to be submitted).

References

- T. Roughgarden, Twenty Lectures on Algorithmic Game Theory, 2016.
- N. Nisan, T. Roughgarden, E. Tardos, and V. Vazirani (editors), Algorithmic Game Theory, 2007. (Book available online for free.)
- R. Myerson, Game Theory: Analysis of conflict, 1991.

Recent papers, and other lecture notes that we will post on the course website.

3 Broad Goals

Goal #1

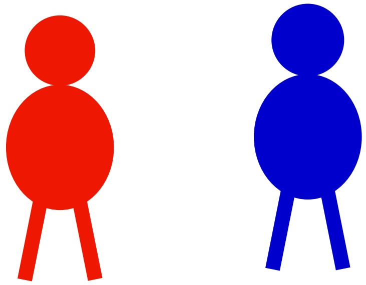
Understand outcomes arising from
interaction of intelligent and self-interested agents.

Games and Equilibria

Prisoner's Dilemma

Two thieves caught for burglary.

Two options: {confess, not confess}

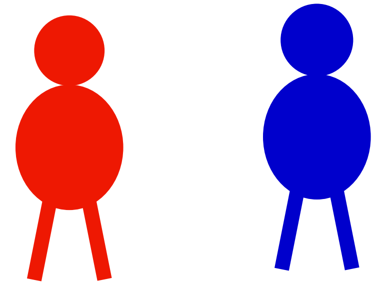


	N	C
N	-1 -1	-6 0
C	0 -6	-5 -5

Prisoner's Dilemma

Two thieves caught for burglary.

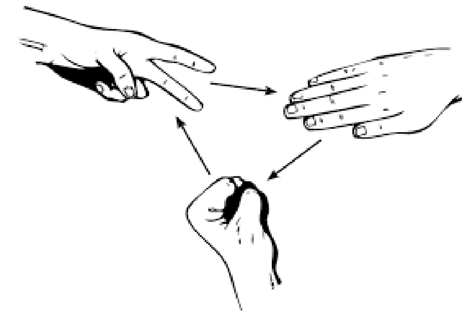
Two options: {confess, not confess}



	N	C
N	-1 -1	-6 0
C	0 -6	-5 -5

Only stable state!

Rock-Paper-Scissors



	R	P	S
R $\frac{1}{3}$	0 0	-1 1	1 -1
P $\frac{1}{3}$	1 -1	0 0	-1 1
S $\frac{1}{3}$	-1 1	1 -1	0 0

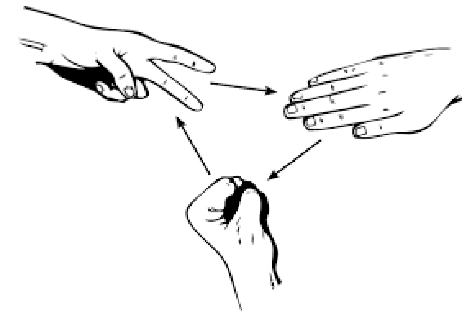
No pure stable state!

Both playing (1/3,1/3,1/3) is a NE.

Nash Eq.: No player gains by deviating individually

Why?

Rock-Paper-Scissors



	R	P	S
R	0 0	-1 1	1 -1
P	1 -1	0 0	-1 1
S	-1 1	1 -1	0 0

No pure stable state!

Both playing (1/3,1/3,1/3) is **the only** NE.

Nash Eq.: No player gains by deviating individually

Why?

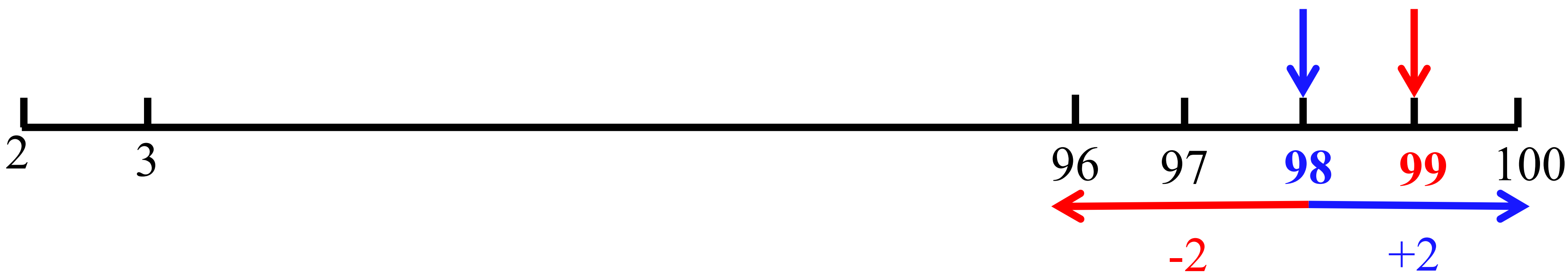
- Finite (normal form) games and Nash equilibrium existence
- Computation:
  - Zero-sum: minmax theorem,
  - General: (may be) Lemke-Howson algorithm
- Complexity: PPAD-complete
- Other equilibrium notions – correlated, markets, security games
- Incomplete information, Bayesian Nash
- Collusion, Core, Nash bargaining

Food for Thought

You and your friend choose a number …

Food for Thought

You and your friend choose a number …



What will you choose?

What if +/- 50?

What are Nash equilibria?

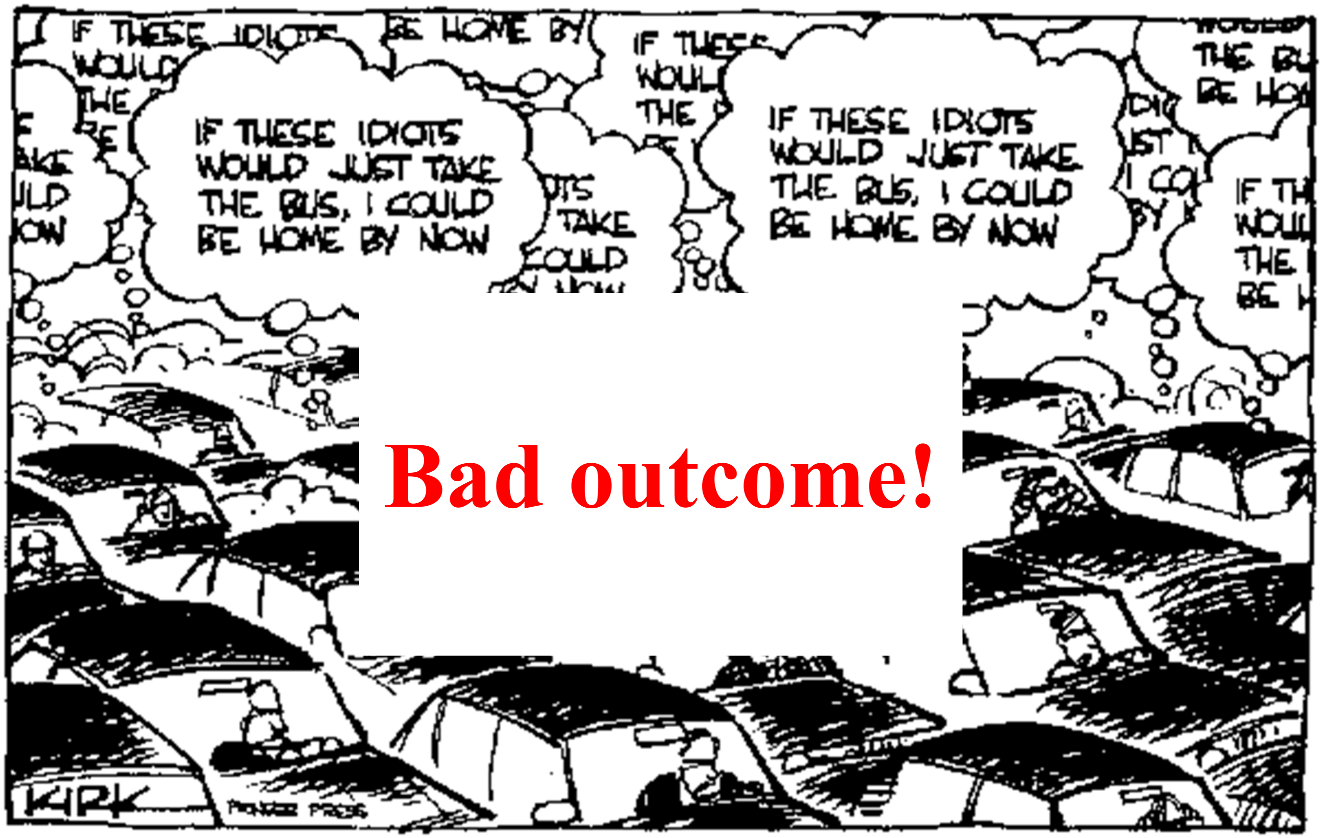
Goal #2

Analyze quality of the outcome arising from strategic interaction, i.e. OPT vs NE.

Price of Anarchy

Tragedy of commons

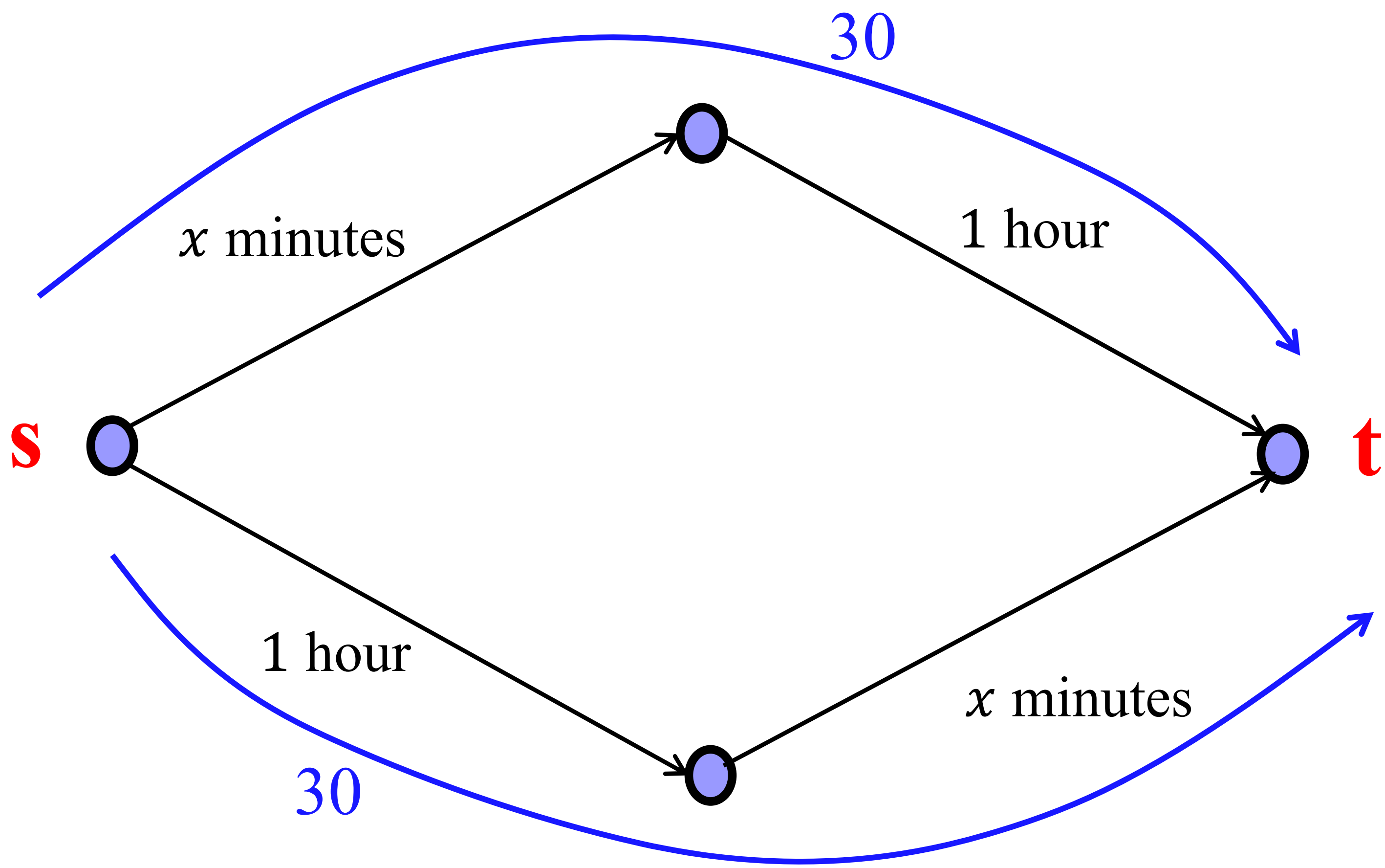
Limited but open resource shared by many.



Stable: Over use => Disaster

Braess' Paradox

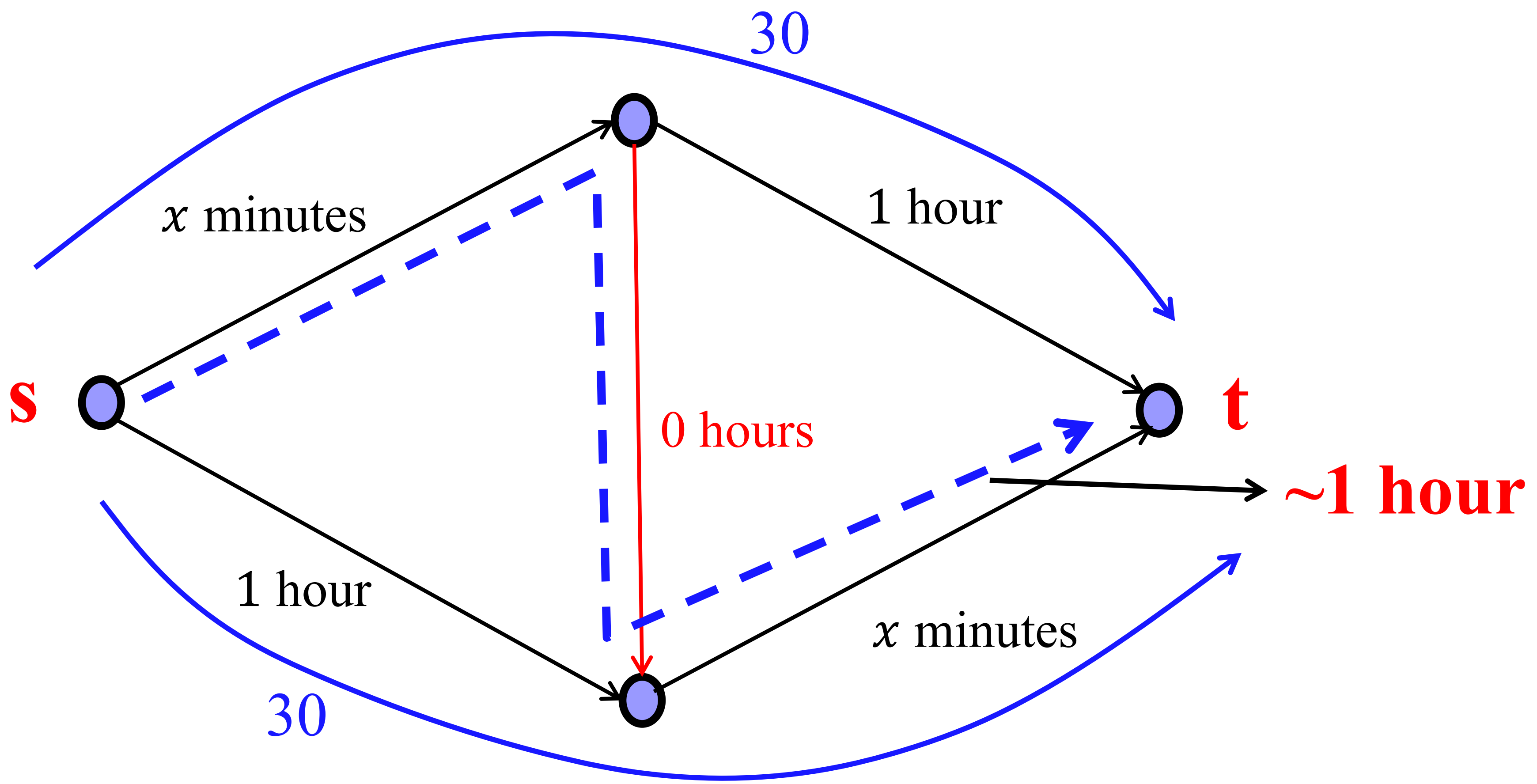
60 commuters



Commute time: 1.5 hours

Braess' Paradox

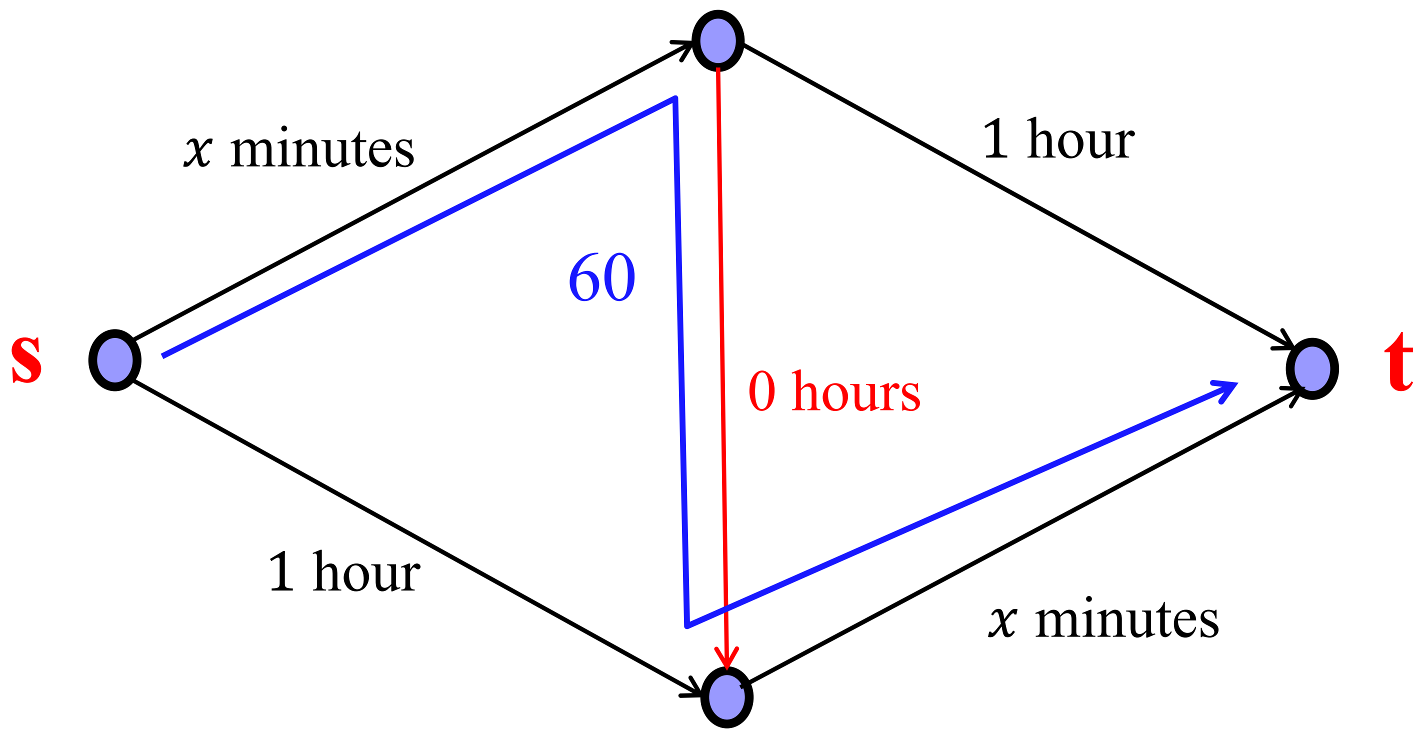
60 commuters



Commute time: 1.5 hours

Braess' Paradox

60 commuters

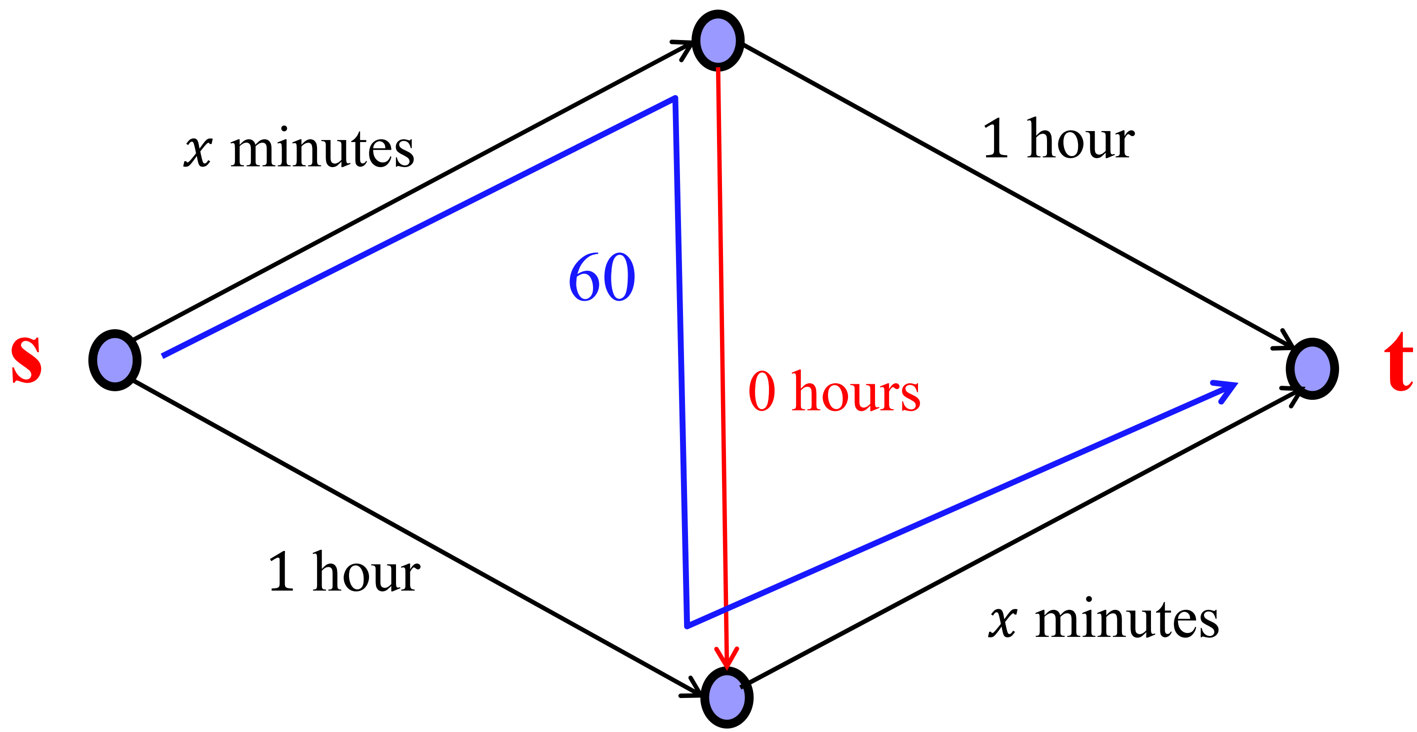


Commute time: 2 hours!

Braess' Paradox in real life

Braess' Paradox

60 commuters



Price of Anarchy (PoA): $\frac{\boldsymbol{worst\ NE}}{\boldsymbol{OPT}} = \frac{\mathbf{2}}{\mathbf{1.5}} = \frac{\mathbf{4}}{\mathbf{3}}$

Can not be worse!

- Network routing games
- Congestion (potential) games
- PoA in linear congestion games
  - Smoothness framework
- Iterative play (dynamics) and convergence

Goal #3

Designing rules to ensure "good" outcome under strategic interaction among selfish agents.

Mechanism Design

At the core of large industries

Online markets – eBay, Uber/Lyft, TaskRabbit, cloud markets

Spectrum auction – distribution of public good. enables variety of mobile/cable services.

Search auction – primary revenue for google!

Tons of important applications

Fair Division – school/course seats assignment, kidney exchange, air traffic flow management, …

Matching residents to hospitals,
Voting, review, coupon systems.

So on …

- MD without money
  - Fair division
    - Divisible items: Competitive equilibrium
    - Indivisible items: EF1, EFX, MMS, Max. Nash Welfare, …
  - Stable matching, Arrow’s theorem (voting)

- MD with money
  - First price auction, second price auction, VCG
  - Generalized second price auction for search (Google)
  - Optimal auctions: Myerson auction and extensions
  - Prophet inequalities and simple auctions
  - Fair MD (may be)

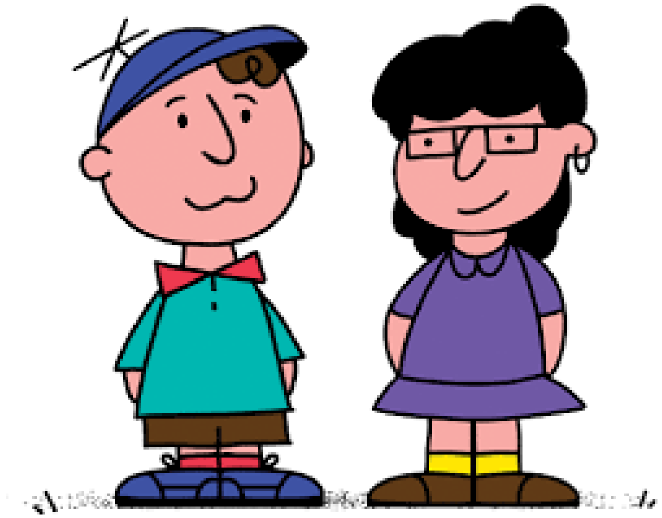
Fun Fact!

Olympics 2012 Scandal

Check out Women’s doubles badminton tournament

Video of the fist controversial match

Example: How to divide fairly?

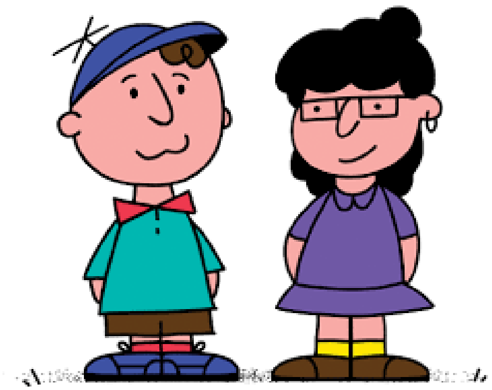


How to divide among the two so that both are happy with their share, and the division seems "fair" to both?

Sol'n: I-Cut-You-Choose

PS: Finds mention in the Bible, in the Book of Genesis (chapter 13).

Example: How to divide fairly?



Sol'n:
I-**Cut-you-Choose**

Envyfree: No one envies other's share

Proportional: Each gets at least half the value
(assuming $v(A \cup B) \leq v(A) + v(B)$, for $A, B \subseteq Cake$)

PS: Finds mention in the Bible, in the Book of Genesis (chapter 13).