Quick Refresh of Auctions

- Selling items of unknown value
- Agents put forth bids to buy items
- With single item, best is second-price/Vickrey auctions

Image from: https://oko.uk/blog/first-price-vs-second-price-auctions
Challenges of multi-item
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One bidder, two items

\[ v_1, v_2 \in \{4, 7\} \]
Challenges of multi-item

One bidder, two items

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In single item, best strategy is to price it at 4

\[ \mathbb{E}[Revenue] = 8 \]
Challenges of multi-item

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Better strategy is to bundle with price at 11
Challenges of multi-item

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In single item, best strategy is to price it at 4

$$\mathbb{E}[Revenue] = 8$$

Better strategy is to bundle with price at 11

$$\mathbb{E}[Revenue] = 8.25$$
Simultaneous Multi-Round Auction
Simultaneous Multi-Round Auction

Each Round
- If agent has highest bid on an item:
  - Doesn’t need to do anything
- Else:
  - Place a higher bid on an item
  - Or drop out

Auction ends when no new bids

Image from: https://www.counterpointresearch.com/us-october-spectrum-auction/
Simultaneous Multi-Round Auction

Goals:

- Reveals information
- Gives bidders ability to respond
Simultaneous Multi-Round Auction

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Problems*:
• Private value ≠ social value

*Notable problems particularly with spectrum auctions, may not occur with other markets, some also applicable to auctions in general
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Image from: https://freebiesupply.com/logos/pagenet-logo/
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Image from: https://www.nwtelco.com/
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Problems*:
- Private value ≠ social value
- Incentive to focus on one item
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  - Bid shading/demand reduction
- Bundles are hard to form

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Combinatorial Clock Auction
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- **Anonymous Bids**
- Round reports include price and demand, no winners
Combinatorial Clock Auction

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- For identical goods (spectrum), **no need for identical lots**
  - Can be specific lots for incumbents
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- Additional final round of closed bids for bundles
Combinatorial Clock Auction

Advantages:
- Easy to auction identical/divisible goods
- No need to determine item winner with every round
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Cons:
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- Bid Sniping
- Computationally Complex
Combinatorial Clock Auction

Payment: Vickrey-nearest-core pricing
  - **Vickrey pricing** introduces truthful bids

\[ b_1(A) = 28 \]
\[ b_2(B) = 20 \]
\[ b_3(AB) = 32 \]
\[ b_4(A) = 14 \]
\[ b_5(B) = 12 \]
Combinatorial Clock Auction

Payment: Vickrey-nearest-core pricing
- Vickrey pricing introduces truthful bids
- Core
  - Set of payment by the winning coalition
  - No other coalition offered to pay more

\[
\begin{align*}
b_1(A) &= 28 \\
b_2(B) &= 20 \\
b_3(AB) &= 32 \\
b_4(A) &= 14 \\
b_5(B) &= 12
\end{align*}
\]
Combinatorial Clock Auction

Payment: Vickrey-nearest-core pricing
- Vickrey pricing introduces truthful bids
- Core
  - Set of payment by the winning coalition
  - No other coalition offered to pay more
- Find closest core prices to Vickrey prices
  - Is unique

\[ b_1\{A\} = 28 \]
\[ b_2\{B\} = 20 \]
\[ b_3\{AB\} = 32 \]
\[ b_4\{A\} = 14 \]
\[ b_5\{B\} = 12 \]
Ausubel Auction

- Anonymous Bids
- Round reports include price and demand, no winners
- Auctioneer records demand for each agent at each price
Ausubel Auction

- Auctioneer records demand for each agent at each price
- Price is cost where rival demand falls below supply
  - Calculated per item and demand and supply is adjusted accordingly

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$s = 4$
**Ausubel Auction**

- Auctioneer records demand for each agent at each price
- Price is cost where rival demand falls below supply
  - Calculated per item and demand and supply is adjusted accordingly

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\[ s = 3 \]
Ausubel Auction

- Auctioneer records demand for each agent at each price
- Price is cost where rival demand falls below supply
  - Calculated per item and demand and supply is adjusted accordingly

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$Alice: 15, 20$

$s = 2$
Ausubel Auction

- Auctioneer records demand for each agent at each price
- Price is cost where rival demand falls below supply
  - Calculated per item and demand and supply is adjusted accordingly

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\[
\text{Alice: } 15, 20, 25 \\
\text{ } s = 1
\]
Ausubel Auction

- Auctioneer records demand for each agent at each price
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Alice: 15, 20, 25
Bob: 25
s = 0
Generalized Second-Price Auction
Generalized Second-Price Auction

- Used in Internet Advertising
Generalized Second-Price Auction

- Used in Internet Advertising
- Single Round Auction
Generalized Second-Price Auction

- Used in Internet Advertising
- Single Round Auction
- Price paid is bid of next lower bid

\[ 100 \cdot 6 = 600 \]
\[ 40 \cdot 1 = 40 \]
Generalized Second-Price Auction

- Used in Internet Advertising
- Single Round Auction
- Price paid is bid of next lower bid
- Untruthful

\[ \alpha_1 = 100 \quad 100 \ (7 - 6) = 100 \]

\[ \alpha_2 = 40 \]
Generalized Second-Price Auction

- Used in Internet Advertising
- Single Round Auction
- Price paid is bid of next lower bid
- Untruthful

\[ \alpha_1 = 100 \quad 100 \ (7 - 6) = 100 \]

\[ \alpha_2 = 40 \quad 40 \ (7 - 1) = 240 \]
Generalized Second-Price Auction

- Used in Internet Advertising
- Single Round Auction
- Price paid is bid of next lower bid
- Untruthful

**Price of Anarchy**
- 2.927 in partial information
- 1.282 in full information in Nash Eq.
Vickrey–Clarke–Groves Auction

- Single Round Auction
- Price paid is **cost to rest of bidders due to participation**

\[
V_N^{M \setminus \{b_i\}} - V_N^{M \setminus \{t_i \}}
\]
Vickrey-Clarke-Groves Auction

- Single Round Auction
- Price paid is **cost to rest of bidders due to participation**

\[
V^M_{N \setminus \{b_i\}} - V^M_{N \setminus \{b_i\}} \quad \alpha_1 = 100
\]
\[
\alpha_2 = 40
\]

\[
(100 \cdot 6 + 40 \cdot 1) - (40 \cdot 6 + 0 \cdot 1) = 400
\]
Vickrey-Clarke-Groves Auction

- Single Round Auction
- Price paid is cost to rest of bidders due to participation
- Truthful
Vickrey-Clarke-Groves Auction

- Single Round Auction
- Price paid is cost to rest of bidders due to participation
- Truthful
- Complex and unintuitive
Vickrey-Clarke-Groves Auction

- Single Round Auction
- Price paid is cost to rest of bidders due to participation
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- Lower revenue than GSP under truthful scenarios
Vickrey-Clarke-Groves Auction

- Single Round Auction
- Price paid is cost to rest of bidders due to participation
- Truthful
- Complex and unintuitive
- Lower revenue than GSP under truthful scenarios and in experiments
Efficiency vs. Equity
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- With **uniform pricing**, price is highest losing bid
Efficiency vs. Equity

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  - May belong to one of the winners
Efficiency vs. Equity

- With uniform pricing, price is highest losing bid
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  - Encourages bid shading/demand reduction
Efficiency vs. Equity

- With **uniform pricing**, price is highest losing bid
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- Uniform Price doesn’t **extract the full value**
Efficiency vs. Equity

- With *uniform pricing*, price is highest losing bid
  - May belong to one of the winners
  - Encourages bid shading/demand reduction
- Uniform Price doesn’t *extract the full value*
- Bid shading also happens with discriminatory auctions
Works Cited


