Optimal Bidding in Large Scale Multi-Item Auctions

Presented by:
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Single vs Multi-Item

- **Single Item**
  - $n$ bidders with valuations $v_i$
  - Second Price Auction $\Rightarrow b_i = v_i$ maximizes utility and social welfare

- **Multi-Item Auction**
  - Collection of $m$ items to be distributed to $n$ bidders
  - $V_{ij}$ for some allocation $j = \{x_1, x_2, \ldots\}$ is agent $i$'s valuation of allocation $j$
  - Introduces lots of additional complexity

(Roughgarden 2013)
Background

- Different types of multi-item auctions
  - Are items identical?
  - Are items divisible?
  - Are items complements/substitutes?
    - $V(A) + V(B) = V(\{A,B\})$?
  - Are items auctioned simultaneously?
  - Sealed or opened?

- Discussion
  - Spectrum Auctions
  - Sponsored Search Auctions
Spectrum Auctions

- Auction system used to sell and assign spectrum resources to various bidders.
- Why Auction?
  - Administrative process, lottery, first come first served etc.
  - Transparent & Fair
  - Maximize Revenues and Utility
  - Economic (McMillan 1995)
- Not identical
- Not divisible
- Simultaneous and Open Bidding
- Not monotone
  - Package Bidding
Simultaneous Ascending Auction

- Introduced by FCC in 1994

<table>
<thead>
<tr>
<th>Bidder 1</th>
<th>Bidder 2</th>
<th>Bidder 3</th>
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<tbody>
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Package Bidding

- More efficient and revenue generating format
- Rise of free rider problem for individual license bids
  - $V1(A) = 4$, $V2(B) = 4$, $V3(AB) = 3$

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<tr>
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<th>B1</th>
<th>B2</th>
<th>B3</th>
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<tr>
<td>B</td>
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<tr>
<td>p1</td>
<td>8-4 = 4</td>
<td>8 - 3 = 5</td>
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<tr>
<td>1-p1</td>
<td>8 - 3 = 5</td>
<td>3-2 = 1</td>
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Collusion

- Bidders working together to reduce auction prices
  - Bid Signaling
  - Retaliation
  - Withdrawal
  - Market Division

- Bid Restricting
- Bidder Identities
- Transparency
- Competition for higher revenue
- Signaling

- Withdrawal Limit
- Reserve Prices
- FCC introduced two round limit
- Market Closing
Reserve Prices Example

- Two identical items, two bidders A and B
- \( V_a \) (one item) = 20, \( V_a \) (both) = 50, \( V_b \) (one) = 17
- Bidder B will put one truthful bid of 17
- No Reserve Price
  - A puts one bid at 20 -> A and B win an item for 0, A's utility is 20
  - A puts one bid at 20 and one at 30 -> A wins both items for 17, utility is 50 - 2(17) = 16
  - Demand Reduction
- Reserve Price = 5
  - A puts one bid at 20 -> A and B win an item for 5, A's utility is 20-5 = 15
  - A puts one bid at 20 and one at 30 -> A wins both items for 17, utility is 50 - 2(17) = 16
Price Forecasting + Budgeting

- For licenses that are substitutes, where should you bid to get the best licenses at the best price? - Price Forecasting (Stanford 2009)

- Exposure
  - Sum of all a bidder’s bids in a given round
  - Total Price * Demand/Supply
  - Equilibrium where Demand = Supply, exposure meets budget
  - Useful for revenue prediction < 10% error

(Bulow, Levin, Milgrom 2009)
Managing Exposure

- Agent bidding on two licenses, \( v_{12} > v_1 + v_2 \)
- Unknown final price \( c_i \) from distribution \( F_i \)
- If opponent on license \( i \) exits, expected utility:

\[
\pi_i(p_1, p_2) = v_i - p_i + Q_j(p_1, p_2). \tag{1}
\]

\[
Q_j(p_1, p_2) = \int_{p_j}^{\max\{b-p_i, p_j\}} \max\{0, (v_{12} - v_i - c_j)\} dF_j(c_j | c_j \geq p_j). \tag{2}
\]

\[
\pi_1(p_1, p_2) = 0 \quad \text{and} \quad \pi_2(p_1, p_2) = 0. \tag{3}
\] (Bulow, Levin, Milgrom 2009)
• Prop1 - Optimal strategy involves raising prices along any path to the unique price pair \((p_{1*}, p_{2*})\) that will solve for (3), drop out at this point if both competitors are still active.

• Prop2 - If the individual bidder for license \(i\) drops out first, continue bidding on license \(j\) until \(p_j = v_{12} - \max\{p_i, v_i\}\)

(Bulow, Levin, Milgrom 2009)
Managing Price Increase

- Holding Back Demand
- Parking
- Jump Bidding
  - Ineffective if done early
  - Risk of overpaying if done late
- SpectrumCo
  - Forecasted final prices
  - Alter rate of price increase
  - Massive market share with a billion dollar discount vs competitor’s prices
Sponsored Search Auctions (SSAs)

- Advertisers bid to be shown alongside results of search engine query
- Multiple advertising slots are available for each search
  - Slots higher on the page are considered more valuable
- Highest bidder gets the highest slot, second-highest bidder gets second slot, etc.
- Auction is run every time a query is made
- Most use generalized second price (GSP) auctions
- For this discussion, assume unlimited budget and full rationality
pizza
Generalized Second Price Auctions

- $n$ bidders compete for $k$ ad slots
  - $n > k$
  - Slot 1 is displayed first, slot $k$ is displayed last
  - $\text{CTR}_1 \geq \text{CTR}_2 \geq \ldots \geq \text{CTR}_k$
- Each bidder $i$ has a private value $v_i$
- Each bidder submits a bid $b_i \leq v_i$
- Bidders are numbered in the descending order of their bids
- Bidder $i$ wins slot $i$ and pays $b_{i+1}$ each time their ad is clicked
- Truth-telling is not a dominant strategy

(Varian 2007)
Truth-Telling in GSPs

- 3 bidders with values per click {10, 4, 2}
- 2 ad slots with CTRs {200, 199}
- If bidder 1 bids truthfully:
  - Payoff = \((v_i - b_{k+1}) \times CTR_k = (10 - 4) \times 200 = 1200\)
- If bidder 1 shades bid to $3:
  - Payoff = \((v_i - b_{k+1}) \times CTR_k = (10 - 2) \times 199 = 1592 > 1200\)
- Bidder 1 increases payout by bidding less than their true value

(Edelman, Ostrovsky, and Schwarz 2005)
Optimal Bidding in SSAs

- Because GSPs are not incentive-compatible, bidders must choose a bidding strategy.
- This results in a continuum of possible Nash equilibria.
- VCG Equilibrium:
  - Possible Nash equilibrium of GSP
  - Player payments are identical to those made if VCG auction was used
  - Cheapest envy-free equilibrium for bidders
- How can we converge to VCG equilibrium over repeated GSPs?
Best-Response Bidding

- Bidder $i$ chooses a bid for the next round that maximizes their utility assuming all other bidders $b_{-i}$ behave the same
  - Utility $u_i = CTR_s(v_i - p_s)$
- $i$ will make a bid $b' \in (p_{s*}(i), p_{s*-1}(i))$ to win slot $s^*$
  - $p_{s*}(i)$ = price that $i$ pays to win slot $s^*$
- Where to bid within this range?
  - Too high: Bidder $i + 1$ will undercut $i$ and force them into a slot with lower utility
  - Too low: Bidder $i - 1$ will out bid
  - Want competitors to pay as much as possible

(Nisan et. al. 2011)
The Balanced Bidding (BB) Strategy

- Bidder $i$ targets the slot $s^*_i$ which maximizes their utility given $b_{-i}$
  - $s^*_i = \arg\max_s \{CTR_s(v_i - p_s(i))\}$
- Bidder $i$ makes the max bid $b'$ that wouldn’t lower utility if they were undercut
  - $CTR_{s^*_i}(v_i - p_{s^*_i}(i)) = CTR_{s^*_{i-1}}(v_i - b')$
- The BB Strategy converges to VCG equilibrium if exactly one random bidder updates their bid after every round
  - Known as the asynchronous bidding model
- What if all bidders simultaneously submit new bids after each round?
  - Requires slightly different strategy called Restricted Balanced Bidding (RBB)

(Cary et. al. 2014)
The Restricted Balanced Bidding (RBB) Strategy

- Bidder $i$ targets the slot $s_i^* \leq s_i$ which maximizes their utility given $b_{-i}$
  - $s_i^* = \arg\max_s \{\text{CTR}_s(v_i - p_s(i))\}$
  - $s_i^*$ must have a CTR no higher than the CTR of $s_i$
- Bidder $i$ makes the max bid $b'$ that wouldn’t lower utility if they were undercut
  - $\text{CTR}_{s_i^*}(v_i - p_{s_i^*}(i)) = \text{CTR}_{s_{i-1}^*}(v_i - b')$
- The RBB Strategy converges to the VCG equilibrium if all bidders change bids after each round
  - Known as the synchronous model
- Both BB and RBB were later proven to be incentive compatible
  - If all bidders use best-response bidding, there is no benefit to deviating

(Cary et. al. 2014)
Collusion in SSAs

- Advertisers have started delegating bidding campaigns to digital marketing agencies (DMAs)
- DMAs often bid on the behalf of multiple advertisers in the same auction
- Coordinating prices between advertisers causes payments to decrease
- Introduces completely different bidding strategies
Collusion in SSAs

- **Reminder:** Bidder $i$ chooses a bid such that getting underbid won’t affect utility
  - $b_i = v_i - (\text{CTR}_i / \text{CTR}_{i-1}) \times (v_i - b_{i+1})$

- **Consider a scenario with 8 bidders and 7 slots**
  - Valuations $v = (12, 10.5, 10.4, 10.3, 10.2, 10.1, 10, 1)$
  - CTRs = $(50, 40, 30.1, 20, 10, 2, 1, 0)$
  - DMA $D$ contains bidders 5 and 6, i.e. $D = \{5, 6\}$

(Decarolis et. al. 2020)
<table>
<thead>
<tr>
<th>Bidder</th>
<th>VCG Equilibrium Bid</th>
<th>Bid After Collusion</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>2</td>
<td>10.11</td>
<td>9.91</td>
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<tr>
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<td>9.67</td>
<td>9.5</td>
</tr>
<tr>
<td>6</td>
<td>9.18</td>
<td>7.94</td>
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<tr>
<td>7</td>
<td>5.5</td>
<td>5.5</td>
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<tr>
<td>8</td>
<td>1</td>
<td>1</td>
</tr>
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Revenue: 55.35  
Revenue: 52.73  

<- Lowered by DMA to benefit bidder 5
Collusion in SSAs

- Reminder: Utility $u_i = CTR_s(v_i - p_s)$
- Bidder 4 undercuts bidder 5 to increase their utility after the DMA lowered the bid of bidder 6
  - Utility of slot 4 = 20(10.3 - 9.5) = 16
  - Utility of slot 5 = 10(10.3 - 7.94) = 23.6
- Bidder 5 then moved up to slot 4, which increased their utility relative to the VCG eq.
  - Utility from VCG eq. = 10(10.2 - 9.67) = 5.3
  - Utility from collusion eq. = 20(10.2 - 9.12) = 21.6
- Resulting allocation is inefficient
• Optimal bidding strategies became difficult to ascertain when these auctions were scaled up
• Both types of auction became susceptible to collusion at large scale
• A clear trade-off emerged between security and efficiency
  • Spectrum auctions are complex and holistic due to the importance and cost of the items being auctioned
  • SSAs are much simpler and intuitive to allow for quick completion and low barrier of entry, making them more susceptible to collusion and inefficiencies
Questions?