Combinatorial Auction (in practice)

- set A of agents
- set M of items \( m = 101 \)

\( V_i \) (i gets something) = 0
\( V_i \) (i gets s) = \( V_i(s) \)

VCG Mechanism: DSIC and SW maximizing

\[ V_i: 2^M \to \mathbb{R}_+ \quad m \sim 100 \times \]

Issue 1: representation of \( V_i \)

- dark about \( V_i(s) \) per need-basis

Issue 2: SW maximizing allocation.

- if \( V_i \)'s are substitutes,
  \[ V_i(A \cup B) \leq V_i(A) + V_i(B) \] \( \Rightarrow \) easy.

- if \( V_i \)'s are complements
  \[ V_i(A \cup B) \geq V_i(A) + V_i(B) \] \( \Rightarrow \) NP-hard.

\[ \times \quad \times \] Approximate, Indirect.

* Sell items separately:

\[ V_i, \ldots, V_m \] \( m \) separate values.
(3): Simultaneous

- similar items, 3-bidders, each unit at least 1.88 M€

Which auction to participate in:

March 2000, Switzerland.

3-sec SIA.

Blocks: 26 MHz 26 MHz
Rev: 121 M 182 M

Take away: Simultaneous SIA.

Q2: Sealed bid / English auction.

- Participate in which auction it wants only one item?

Coordination issues - If put 28 multiple then how to bid in order to own one item?

1980 New Zealand.

Blocks: Several block of similar size (850 MHz!)

Auction Rev: 36 M!

Highest bid: $100,000 $7 M

Second-H. bid: $6 $5 K
First price. Didn't help.

Take away 2: English auction.

Simultaneous Ascending Auction (SAA)
- each item sold separately
- English Ascending Price auction for each

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2 items 2 bidders (each acts only once)
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<table>
<thead>
<tr>
<th>Rounds</th>
<th>V₁ = 10</th>
<th>V₂ = 8</th>
<th>V₃ = 5</th>
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<td>(0.1, 0.1)</td>
<td>(0.5, 0.5)</td>
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<td>(0.5, 2)</td>
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<td>(3, 2)</td>
<td>(4, 4.0)</td>
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<td>(4, 5.1)</td>
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</table>
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VCG outcome.

"Pros"
- Resolves coordination issues
- Agents need not know their values upfront.
- "works well" (if all similar blocks go for similar price)

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X X X
- "Woes ween"
  - Similar blocks go for similar price
  - No outside transaction / at similar price sell.
  - Price discovery: correlation between mid-auction (winner, price) & final (winner, price)
  - Exceeds projected Rev.

- "Cons"
  - Demand Reduction.
    \[ V_1(A) = V_1(B) = 10 \quad V_1(AB) = 20 \]
    \[ V_2(A) = V_2(B) = V_2(AB) = 8. \]

(SAA) First bidder wins both
- (hurtful) & pays \( 8 + 8 = 16 \)
  \[ U_1 = 20 - 16 = 4 \]

(SAA) First bidder goes only for B \( \Rightarrow \) pay minimal
- (unhurtful): Then second bidder will go for A. \( \Rightarrow \)
  \[ U_1 = 10 - 2 \gg 4. \]

- Exposure Problem.
  \[ V_1(AB) = 100 \]
  \[ V_1(AB) = 100 \]
  \[ V_2(A) = V_2(B) = V_2(AB) = 75. \]

Either I wins both at price of 75 each
- (hurtful) \[ U_1 = 100 - 150 < 0. \]

Or I drops out at price of (50, 50)
OR 1 drops out at price $50, 2
then 2 wins both & pays $100!

$v_2 = 75 - 100 < 0$

* Soln*: package bidding.

1. Fixed packages.

   $A, B, C, D, AB, CD.$

   What if a bidder wins $AC$?

2. First run SAT: Round 1

   - limited package bidding: Round 2.

3. Now.

   Both package & single bids. \( \text{} \geq 2022 \).

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* Reverse Auction \( \sim 2014 \).
Buy out + Repack the remaining in 40-50 MHz band.

- Intuitively buy from low-welfare agents
  sell to high-welfare agents.

- Repack is a relatively NP-hard problem.
  Graph-coloring.

⇒ Direct Mechanism:
  bidder \( i \) specifies \((b_i, r_i)\)
  bid.

⇒ We say \( W \subseteq A \) is a feasible set
  if \( A \setminus W \) can be repacked into
  available range.

1. \( W = A \) is feasible because \( A \setminus W = \emptyset \), so nothing to repack.
   \( \Rightarrow \) \( W \) is feasible.
1. \( W = H \setminus w \) to repack.

2. While \( \exists w \) s.t. \( W \setminus w \) is terrible

3. Resume one such \( i \) from \( w \).

3. Return \( w \).

( depends on bid, storage, bid/capita )

\[ \downarrow \]

give score to agents

\[ \downarrow \]

pick highest score agent to drop from \( w \).

If score increases and increase in bid

Then the allocation rule is Monotone \( \downarrow \) by Myerson's payment.

DSIC Mechanism.