Reverse Auction & Routing Games.

Last lec: Spectrum Auctions. (FCC)

- Item = \((\text{location, freq band})\) ⇒ Heterogeneous items ⇒ Combinatorial auction.

⇒ VCG is not applicable.

- Simultaneous Ascending Auctions (sell similar items separately in English auction format)

  * Package bidding.

- Since 2014: FCC does two step process:

  1. **Step 1:** Reverse auction (buy spectrum)
  2. **Step 2:** Forward (sell)

  Based on “Desired” & who all are willing to sell/liquidate

  At the location: “Available” “Free Up”

  20 60 100 MHz.

Step 1:

- A: set 25 agents
  - Agent \(i\) has value \(v_i\) for her spectrum. (Private)
  - \(b_i\): bid or agent \(i\)

  \(v_i\): range if owns.

⇒ If we buy out \(S \subseteq A\) then we have to pack \((A\setminus S)\) in the “available” range.
We say \( S \) is a \(^\text{"feasible"}^\) set of winners if \( A \setminus S \) can be packed in the \(^\text{"available"}^\) range.

**Algorithm (Reverse Auction):**

1. Init. \( S = A \). (\( \Rightarrow \ A \setminus S = \emptyset \) \& hence \(^\text{"feasible"}^\))

2. While \( \exists i \in S \) s.t. \( S \setminus \{i\} \) is \(^\text{"feasible"}^\)

   Remove one such \( i \) from \( S \) \( \Rightarrow \emptyset \)

End while

3. Declare \( S \) as winners.

\( \emptyset \) is underdetermined, how to ensure some one allocation rule?

\( \Rightarrow \) If all agents are \(^\text{"equal"}^\) then remove the \(^\text{"highest bidder"}^\)

\( \Rightarrow \) Per-capita \(^\text{"highest bid"}^\)

\( \Rightarrow f(\text{bid}, \text{sec, etc-type} \& \text{specrum}) \).

\( \emptyset \) Re-packing \( A \setminus S' \) \( (S' = S \cup \{i\}) \): packing + clearing.

\( \emptyset \) This graph \( k \)-colorable?
SAT solvers.

**B.** Braess’s Paradox

\( \text{cost}^P(50, 50) = \frac{50}{100} + 1 = 1.5 \)

\( \text{cost}(50, 50) = 1.5 \times 100 = 150 \)

\( \text{cost}^P(s-a-b-t) = \frac{50}{100} + \frac{5}{100} = 1.57 \)

\( \text{cost}(s-a-b-t) = \frac{50}{100} + \frac{5}{100} = 1.57 \)

\( \text{cost}^P(\text{NE}) = \frac{100}{100} + \frac{100}{100} = 2 \)

\( \text{cost}(\text{NE}) = 200 \)

\( \text{cost}(\text{OPT}) = 150 \).

What is \( \text{OPT} \)?

\( (50, 50) \)

"Price vs. Anarchy" = \( \frac{\text{cost}(\text{cost NE})}{\text{cost}(\text{OPT})} = \frac{1.57}{1.5} \approx \frac{1}{3} \)

A. Pigro N/w.

\( \text{cost}(x, 1-x) = x \cdot x + (1-x) \cdot 1 \)

\( = x^2 + 1 - x \).

**NE:** No "incentive to deviate" slow can change their path and reduce their cost.

0.1 vs 0.9

\( \text{cost} 0.1 \) vs \( \text{cost} 1 \).
\[ NE = (1, 0), \quad \text{cost}(1, 0) = 1 \]

\[ \text{Opt} = \arg \min_x \ (x^p + (-x)) \]

\[ \frac{d}{dx} \text{cost} = 2x - 1 = 0 \quad \Rightarrow \quad x = \frac{1}{2} \]

\[ \text{Opt} = \left( \frac{1}{2}, \frac{1}{2} \right) \quad \text{cost} (\text{Opt}) = \frac{1}{4} + 1 - \frac{1}{2} = \frac{1}{4} + \frac{1}{2} = \frac{3}{4} \]

\[ p \rightarrow \frac{\text{cost}(NE)}{\text{cost}(\text{Opt})} = \frac{1}{\frac{3}{4}} = \frac{4}{3} \]

\[ \text{lo-incidence?} \]

\[ \begin{align*}
\text{cost}(x, 1-x) &= x \cdot x^p + (1-x) \cdot 1 \\
&= x^{p+1} + 1 - x \\
\end{align*} \]

\[ NE = (1, 0), \quad \text{cost}(NE) = 1 \]

\[ \text{Opt} = \arg \min \ x^{p+1} - x \]

\[ \frac{d}{dx} \text{cost} = (p+1) x^p - 1 = 0 \quad \Rightarrow \quad x = \left( \frac{1}{p+1} \right)^{\frac{1}{p}} \]

\[ \text{cost} (\text{Opt}) = \lim_{p \to 0} \left( \frac{1}{p+1} \right)^{\frac{1}{p}} + 1 - \left( \frac{1}{p+1} \right)^{\frac{1}{p}} \]
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1. \[ r_1 \rightarrow r_1 \]
2. \[ 0 \rightarrow 1 \]
3. \[ 0 + 1 - 1 = 0 \]

\[
\text{PoA} = \frac{\text{cost(NE)} = x}{\text{cost(LP)} = 0} \rightarrow \infty
\]

**Conclusion:**
- Definitely the degree of the cost-function matters.
- Does the capacity of m/w matter?
- Goal: No!!

**Set up**
- A directed m/w \( G = (V, E) \)
- Special nodes in \( V \).
- \( 2 \)-unit flow has to go from \( s \) to \( t \).

For each edge \( e \in E \), cost function \( c_e : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \)

\[ c_e \in C \]

Non-dec, convex, continuous.

Then: Given a class \( C \) of cost functions, among all m/w's with edge costs from \( C \), "fishou-like" m/w has the worst PoA.