

★ Spectrum Auction :

item = (band, geographic area / location)

"  
freq range

$M = \text{set of items.}$

★ Landing slots on airports.

item = (time, airport)

$m = |M|$

Assumption: Every agent only cares about what they get & not what others get.

$V_i(\text{i getting nothing}) = 0$

$V_i(\text{i getting set } S) = V_i(S)$

$\sum_i V_i(b_i(w))$

★ VCG is "the" DSIC mechanism. → Outcome = arg max  $w \in \Omega$

★ Value  $V_i: 2^M \rightarrow R_+$

Issue 1: Representation.

Suppose ask about  $V_i$  per seed basis

Issue 2: S.W. max is NP-hard.

↳ Substitutes then it's easy  
 $A, B \subseteq M, V_i(A \cup B) \leq V_i(A) + V_i(B)$   
 ↳ Complement then it's hard

$V_i(A \cup B) \geq V_i(A) + V_i(B)$



★ sell every item separately.

↳ multi-item auctions.

★ sell every item separately.

1 - - - m  
v<sub>11</sub> - - - v<sub>1m</sub>

m separate single-item auctions.

Mistake 1: Seq. SIA

List: Simultaneous.

March 2000 Switzerland.  
3-seq SIA

Blocks: 28MHz 28MHz 56MHz

Rev: 184M ↔ 121M 55M!  
not acceptable

Q2: Sealed bid vs inc/dec price

- participate in only one!

- participate in more but conservatively

(coordination issue)

Bidders' options are (if they want only one item).

1990 New Zealand.

Block: several similar blocks. (worth 250M).

Auc. Rev: 36M!

Highest bid \$100,000

\$7M

↕

second " " \$6

\$5K.

↳ First price sealed bid simultaneous.



# Simultaneous Ascending Auction. (SAA)

- each item sold separately
- English auction for each
- fixed time for each round.

2-similar items  $v_1 = 10$       3-bidders  $v_2 = 8$        $v_3 = 5$

Rounds			
1.	(0.1, 0.1)	(1, 1)	( <u>2</u> , <u>2</u> )
2.	( <u>2</u> , 0.1)	(1, <u>2</u> )	(2, 2)
3.	(2, 0.1)	(1, 2)	( <u>3</u> , <u>4</u> )
⋮	⋮	⋮	⋮
6.	( <u>5</u> , 0.1)	(3, 4)	(4, <u>5</u> )
7.	(5, 0.1)	( <u>5</u> .1, 4)	(4, <u>5</u> )
8.	(5, <u>5</u> .1)	( <u>5</u> .1, 4)	(4, 5)

} → VCG outcome.

## ★ Pros:

- Removes the coordination issue
- Agents need not know valuation upfront
- "works well":
  - Similar items go for similar price
  - No outside transactions / outside price

- No outside transactions / at same price
- Price Discovery: winner + price at bid-auction are correlated w/ winner + price at the end
- Exceeds the projected Rev.

★ "Loss"

→ Demand Reduction (w/ substitutes & complementary valuations)

$$v_1(A) = v_1(B) = 10, \quad v_1(AB) = 20$$

$$v_2(A) = v_2(B) = v_2(AB) = 8$$

VCG: Give both to agent 1      s.w. = 20

$$\text{Payment}_1 = 8$$

SAA: Give both to agent 1  $\Rightarrow U_1 = 20 - 16 = 4$   
 (truthful)       $\text{Payment}_2 = 8 + 8 = 16$

SAA (out-truthful)  
 agent 1 bids only on (A)  $\rightarrow$  get items at low price  
 agent 2 bid on B  $\rightarrow$  reserve price.

$$U_1 = 10 - \underline{\epsilon} \Rightarrow 20 - 16 = 4$$

→ Exposure Problem (complements)

$$v_1(A) = v_1(B) = 0, \quad v_1(A, B) = 100$$

$$v_2(A) = v_2(B) = v_2(AB) = 75$$

$$v_1(A) = 75$$

$$v_2(A) = v_2(B) = v_2(AB) = 75$$

(75 + 75)

SAA: (truthful) Either 1 will win  $\{A, B\}$  pay 150!  
 or 1 drops out at (50, 50) then  
 2 wins both & pays 100!

★ Improvements in SAA to eliminate exposure problem.

① First run SAA as single items.  
 Then a round of package bidding

② Fixed packages  
 ABCD EFGH  
 cost (CDEF) ?

③ Allow fixed # of packaged bids as you choose  
 as fixed size.

