Designing “rules of the game” to achieve desired outcomes.

Auctions
- Single seller / auctioneer.
- Sing item to sell: (indivisible)
- Set of agents / bidders / players: 1, 2, m
- Agent i has value $v_i$ for item. Private information

Sealed Bid Auction:
1) Auctioneer solicits bids from the agents
   Agent i submits bid $b_i$ in a “sealed envelope”
   $b_i$ need not be $v_i$

2) After looking at the bids auctioneer decides
   “Winner” & “Payment”

   - Winner = The highest bidder
   - Payment $p_i = \min(b_i, v_i)$
   - Revenue of auctioneer: $\sum_i v_i - \sum_i p_i$ (maximize)
   - Goal of auctioneer: give item to agent i
   - Agent i's utility
      $\delta v_i$ agent $i = v_i - p \geq 0 \text{ if } i \text{ is the winner}$

Utility of agent $i = v_i - p \geq 0 \text{ if } i \text{ is the winner} = 0 \text{ o.w.}$

<table>
<thead>
<tr>
<th>Winner = Highest bidder</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payoff Rules</td>
<td>$v_i$</td>
<td>1200</td>
<td>200</td>
</tr>
<tr>
<td>Payoff Rules</td>
<td>$v_i$</td>
<td>170</td>
<td>200</td>
</tr>
<tr>
<td>Payoff Rules</td>
<td>$v_i$</td>
<td>900</td>
<td>170</td>
</tr>
</tbody>
</table>

Pay the highest bid = FP Bid=$p_i$
Pay the highest bid = FP Bid.
Pay the second highest bid = SP Bid.

* First price: Highest bidder wins, pays her bid.

\[ n = 2, \quad \text{suppose } b_2 = \frac{v_2}{2}, \quad v_1, v_2 \sim U[0, 1] \]

If 1 bids \( b_1 \), then

\[ U(b_2) = (v_1 - b_1) \cdot P(b_1 = b_2) + 0 \cdot P(b_1 < b_2) \]

\[ = (v_1 - b_1) \cdot P\left( \frac{v_2}{2} \leq b_1 \right) \]

\[ = (v_1 - b_1) \cdot P\left( v_2 \leq 2b_1 \right) \]

\[ = (v_1 - b_1) \cdot 2b_1 \]

**approx**

\[ b_1 \in [0, 1] \]

\[ \frac{dU}{db_1} = 2v_1 - 4b_1 = 0 \]

\[ b_1 = \frac{v_1}{2} \]

Similarly, if \( b_1 = \frac{v_1}{2} \), then \( b_2 = \frac{v_2}{2} \) is best for player 2.

\( \left( \frac{v_1}{2}, \frac{v_2}{2} \right) \) is a NE.

* In general: \( n \) bidders \( v_1, \ldots, v_n \sim U[0, 1] \)

Suppose fix \( b_i = \frac{(n-1)v_i}{n} \) for \( i \geq 1 \).

Then we can show that \( b = \frac{(n-1)v_i}{n} \) is the
Here we can show that $b_i = (n-1) \frac{v_i}{n}$ is the best strategy for player $i$.

$\left( \frac{n-1}{n} v_1, \ldots, \frac{n-1}{n} v_n \right)$ is a NE.

- $v_1, \ldots, v_n$ are drawn from a complex distribution.

Second Price: Highest bidder wins & pays the second highest bid.

Then (Vickrey '61): For each agent $i$, $b_i = v_i$ is the best strategy no matter what others bid.

$b_i = v_i$ is the dominant strategy for $(V_i, \ldots, V_n)$ is the DSE.

PF: Fix an agent $i$ if bids $b_k + k + 1$. Agent $i$ bids $b_i$, care I: $i$ wins $\Rightarrow u_i = V_i - \max_{k \neq i} b_k$ payoff is independent of your bid. care II: $i$ loses $\Rightarrow u_i = 0$

$b_i = V_i$ is best

\[ b_i \Rightarrow u_i = 0 \]

$b_i = v_i$ is best
\( b_i = v_i \Rightarrow \text{best} \)

\( b_i \Rightarrow v_i = 0 \)

Same as \( k(b, v_i) \)

\( E \text{Bay} = \text{English auction} \)

- Start with very low price, other probably everyone wants to buy. And keep increasing until only one remains

\[
\begin{align*}
V_1 & \geq V_2 \geq \cdots \geq V_n & \text{Price} \\
\frac{\text{all } n \text{ agents}}{n_i \text{ agents}} & \geq P_0 \\
\frac{\text{agent } p_k}{\cdots} & \\
\end{align*}
\]

\( \Rightarrow \text{essentially fixed price auction} \)

\( V_1 \geq \cdots \geq V_n \)

\( P_0 \geq \)