Bargaining

Thursday, September 30, 2021 1:53 PM

* Efficiency
* Symmetry:

} Desired Properties

$X = \{(x_1, x_2) \geq 0 \mid x_1 + x_2 = 1\}$

$D = (0,0)$

splitting a dollar eg.

* Focus on 2-player setting.

$X = \{$set of possible outcomes of the bargaining$\}$

D = Disagreement point if bargaining fails

$D \in X$.

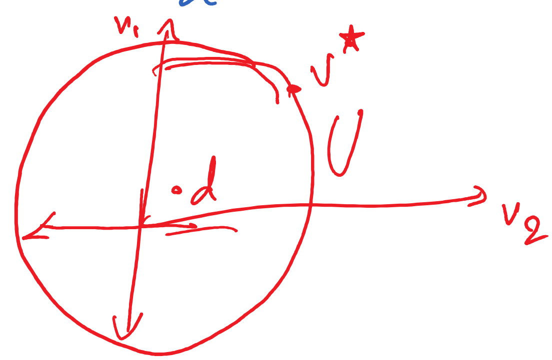
$U = \{(v_1, v_2) = (u_1(x), u_2(x)) \mid x \in X\}$, $d = (u_1(D), u_2(D))$

utility functions of agents

$d \in U$.

Assumptions: U is convex, compact.

(U, d) is a bargaining instance



* Good Properties of Bargaining

$f: B \longrightarrow U$

B: (U, d)

$f(U, d) = v^*$

(0) $v^* \geq d$

(PO) Pareto-optimal

(Sym) Symmetry: if $d_1 = d_2$ & U is symmetric ($\equiv$ $(v_1, v_2) \in U \iff (v_2, v_1) \in U$)

Then $v_1^* = v_2^*$

SI $(\alpha_1, \alpha_2) > 0, \quad (\beta_1, \beta_2)$

$$U' = \{(\alpha_1 v_1 + \beta_1, \alpha_2 v_2 + \beta_2) \mid v_1, v_2 \in U\}$$

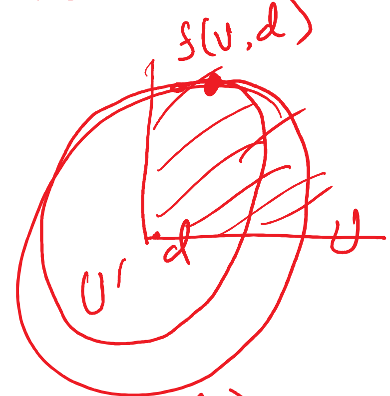
$$d' = (\alpha_1 d_1 + \beta_1, \alpha_2 d_2 + \beta_2)$$

Then $f_i(U', d') = \alpha_i f_i(U, d) + \beta_i \quad i = 1, 2$

IIA Independent of Irrelevant Alternatives.

$U' \subseteq U$, if $f(U, d) \in U'$

Then $f(U', d) = f(U, d)$



Nash's function

$f^N : B \to U$ $\quad$ max: $\log(v_1 - d_1) + \log(v_2 - d_2)$

$$\boxed{f^N(U, d) = \underset{(v_1, v_2) \in U, v \geq d}{\arg\max} (v_1 - d_1)(v_2 - d_2)}$$

Observations: $\boxed{f^N(U, d) \text{ is unique.}}$

Thm: f satisfies 4 axioms iff $f = f^N$

Pf: $(\Leftarrow)$ f^N satisfies all 4 axioms.

PO ✓

Symm

$\max\limits_{(v_1, v_2) \in U} : (v_1 - d)(v_2 - d)$ $\quad v \geq d)$

suppose not, $(v_1^*, v_2^*) = f^N(U, d)$

$(v_2^*, v_1^*) \in U$

$OPT = (v_1^* - d)(v_2^* - d) = (v_2^* - d)(v_1^* - d)$

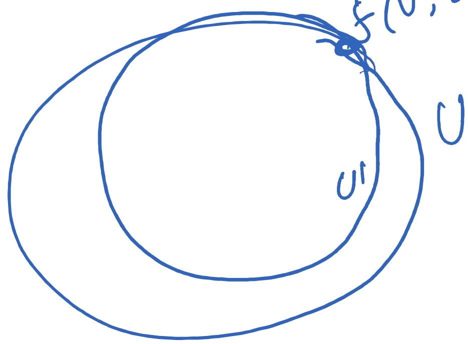
Both (v_1^*, v_2^*) & (v_2^*, v_1^*) are OPT X!

iff $v_1^* = v_2^*$ ($\because$ f^N

SC Exercise

IIA

$f^N(U, d) = f^N(U', d)$



($\Rightarrow$) TPT $f(U,d) = f^N(U,d)$ $\forall (U,d)$.

Use S.I. to reduce to $U', d' = (0,0)$

s.t. $\boxed{f^N(U', 0) = \left(\frac{1}{2}, \frac{1}{2}\right)}$

$\left(\frac{1}{2}, \frac{1}{2}\right) \leftarrow (v_1^*, v_2^*) = f^N(U, 0)$

$(0, 0) \leftarrow (d_1, d_2)$

} Computing α, β Exercise.

It suffices to show that $f(U', 0) = \left(\frac{1}{2}, \frac{1}{2}\right)$

(S.I) $\Leftrightarrow$ $f(U,d) = f^N(U,d)$

$f(U'')$

claim:

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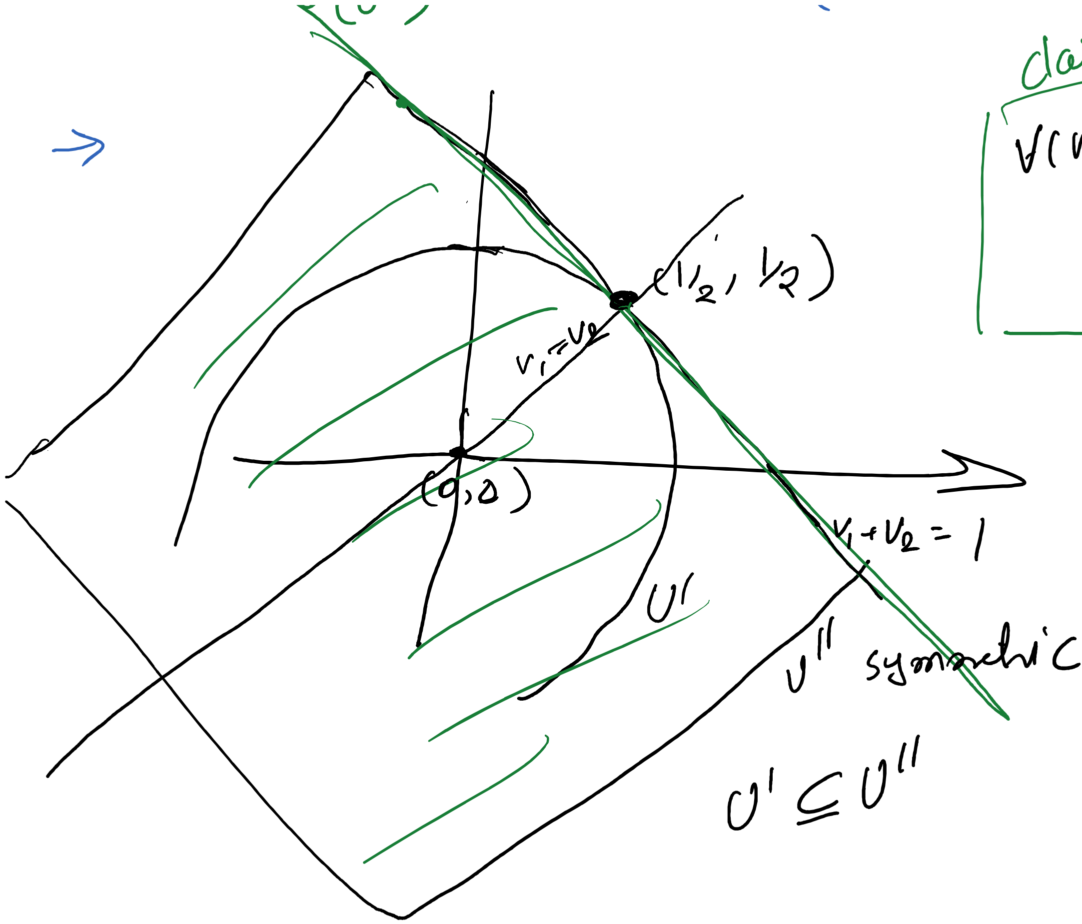
$\forall (v_1, v_2) \in U'$

$v_1 + v_2 \le 1$



$U' \subseteq U''$

$f(U'', (0,0)) = (\frac{1}{2}, \frac{1}{2})$

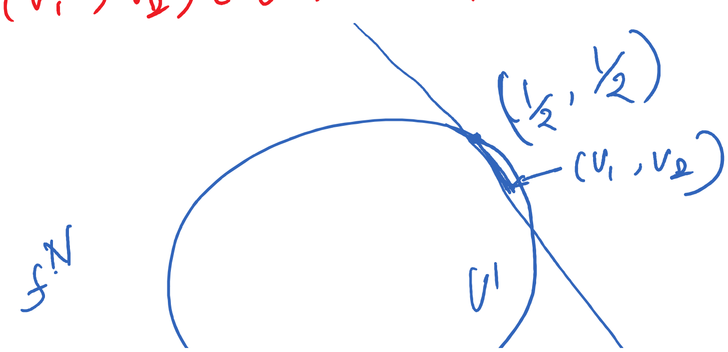
PO, Sym

$\Downarrow$ $U' \subseteq U''$, IIA

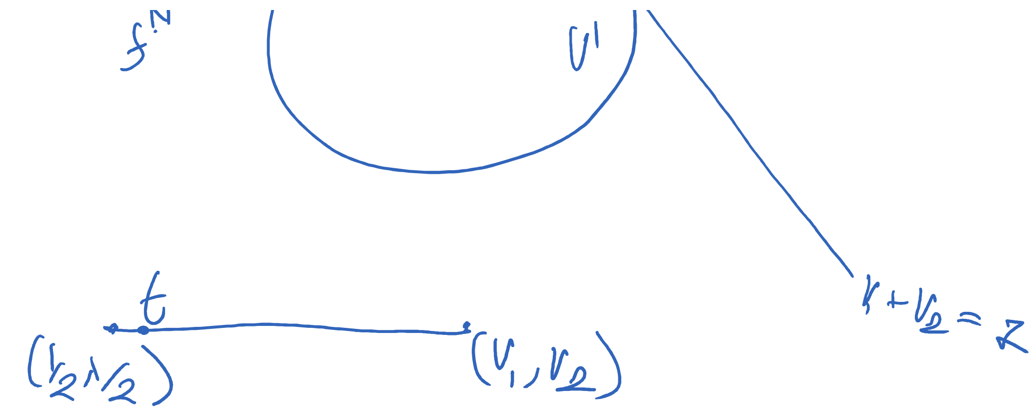
$f(U', (0,0)) = (\frac{1}{2}, \frac{1}{2})$

Claim: $\forall (v_1, v_2) \in U'$, $v_1 + v_2 \le 1$

Pf:



s.t. $v_1 + v_2 > 1$



$t = (1-\lambda)\left(\frac{1}{2}, \frac{1}{2}\right) + \lambda (v_1, v_2)$ tiny λ

$$\underline{(t_1 - 0)(t_2 - 0)} = \left((1-\lambda)\frac{1}{2} + \lambda v_1\right) \cdot \left((1-\lambda)\frac{1}{2} + \lambda v_2\right)$$

$$= (1-\lambda)^2 \frac{1}{4} + \frac{\lambda(1-\lambda)}{2}(v_1 + v_2) + \lambda^2 v_1 v_2$$

$$> \underbrace{(1-\lambda)^2 \frac{1}{4}}_{\downarrow \, 1/4} + \frac{\lambda(1-\lambda)}{2} + \lambda^2 v_1 v_2$$

$\lim_{\lambda \to 0}$

$f^N(U', 0) = \left(\frac{1}{2}, \frac{1}{2}\right)$

max value $\left(\frac{1}{2} - 0\right)\left(\frac{1}{2} - 0\right) = \frac{1}{4}$

$$(t_1 - 0)(t_2 - 0) > \left(\frac{1}{2} - 0\right)\left(\frac{1}{2} - 0\right) \quad !$$