CS 580

Algorithmic Game Theory

Instructor: Ruta Mehta
TA: Rucha Kulkarni
Game Theory

Multiple self-interested agents interacting in the same environment

Deciding what to do.

Q: What to expect? How good is it? Can it be controlled?
Game of Chicken (Traffic Light)
Algorithmic Game Theory

**AGT**, in addition, focuses on designing efficient algorithms to compute solutions necessary to make accurate prediction.
- **What to expect**
  
  Research-oriented Course

  - Exposure to key concepts and proof techniques from AGT
  - Explore research problems and novel questions

- **What is expected from you**

  - Pre-req: Basic knowledge of linear-algebra, linear programming, probability, algorithms.
  - Energetic participation in class
  - Research/Survey Project (individually or in a group of two).
Instructor: Ruta Mehta (Me)

TA: Rucha Kulkarni

Office hours:

- Ruta: Wed 2:30-3:30pm in Siebel 3218
- Rucha: Mon 2:30-3:30pm in TBD
Useful links

- Webpage: https://courses.engr.illinois.edu/cs580/fa2021

- Piazza Page: piazza.com/illinois/fall2021/cs580

Check webpage/piazza at least twice a week for the updates.

HW0 is already posted!
Grading:

- 3 homeworks – 30% (10,10,10)
- Research/Survey Project – 45%
  - Work – 20%
  - Presentation – 12.5%
  - Report – 12.5%
- Final Exam – 22%
- Class participation – 3%

HW0 is for self-study and carry no points.
References


- N. Nisan, T. Roughgarden, E. Tardos, and V. Vazirani (editors), Algorithmic Game Theory, 2007. (Book available online for free.)


Recent papers, and other lecture notes that we will post on course website.
3 Broad Goals
Goal #1

Understand outcomes arising from interaction of intelligent and self-interested agents.

Games and Equilibria
Prisoner’s Dilemma

Two thieves caught for burglary.
Two options: {confess, not confess}

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Only stable state
Rock-Paper-Scissors

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No pure stable state! Both playing \((1/3, 1/3, 1/3)\) is the only NE.

**Nash Eq.:** No player gains by deviating individually. Why?
- Normal form games and Nash equilibrium existence

- Computation:
  - Zero-sum: minmax theorem,
  - General: (may be) Lemke-Howson algorithm

- Complexity: PPAD-complete

- Other equilibrium notions – markets, security games

- Incomplete information, Bayesian Nash

- Collusion, Core, Nash bargaining
Tragedy of commons

Limited but open resource shared by many.

Bad outcome!

Stable: Over use => Disaster
Goal #2

Analyze quality of the outcome arising from strategic interaction, i.e. OPT vs NE.

Price of Anarchy
Braess’ Paradox

60 commuters

Commute time: 1.5 hours
Braess’ Paradox

60 commuters

Commute time: 1.5 hours
Braess’ Paradox

60 commuters

Commute time: 2 hours
Braess’ Paradox

60 commuters

Price of Anarchy (PoA): \( \frac{\text{worst NE}}{\text{OPT}} = \frac{2}{1.5} = \frac{4}{3} \)

Can not be worse!
- Network routing games
- Congestion (potential) games
- PoA in linear congestion games
  - Smoothness framework
- Iterative play and convergence
Goal #3

Designing rules to ensure “good” outcome under strategic interaction among selfish agents.

Mechanism Design
At the core of large industries

Online markets – eBay, Uber/Lyft, TaskRabbit, cloud markets

Spectrum auction – distribution of public good. enables variety of mobile/cable services.

Search auction – primary revenue for google!
Tons of important applications

Fair Division – school/course seats assignment, kidney exchange, air traffic flow management, …

Voting, review, coupon systems.

So on …
- **MD without money**
  - **Fair division**
    - Divisible items: Competitive equilibrium
    - Indivisible items: EF1, EFX, MMS, Max. Nash Welfare, …
  - **stable matching, Arrow’s theorem (voting)**

- **MD with money**
  - First price auction, second price auction, VCG
  - Generalized second price auction for search (Google)
  - Optimal auctions: Myerson auction and extensions
Fun Fact!

Olympics 2012 Scandal
Check out Women’s doubles badminton tournament

Video of the fist controversial match
Food for Thought

You and your friend choose a number ...
Food for Thought

You and your friend choose a number …

What will you choose?    What if $+/- 50$?

What are Nash equilibria?