

Problem Set #2

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Due: *Thu., 2026-09-24 12:00pm*

All problems are of equal value.

1. (Sipser #7.21) Let $\text{DOUBLESAT} = \{\langle \varphi \rangle : \varphi \text{ has at least two satisfying assignments}\}$. Show that DOUBLESAT is NP-complete.
2. (Sipser #7.39) In the proof of the Cook-Levin theorem, a window is a 2×3 rectangle of cells. Show why the proof would have failed if we had used 2×2 windows instead.
3. (Sipser #7.17) Show that, if $P = NP$, then every language $A \in P$, except $A = \emptyset$ and $A = \Sigma^*$, is NP-complete.
4. (Sipser #8.12) Show that TQBF restricted to formulas where the part following the quantifiers is in conjunctive normal form is still PSPACE-complete.
5. (Sipser #8.22)
 - (a) Let $\text{ADD} = \{\langle x, y, z \rangle : x, y, z > 0 \text{ are binary integers and } x + y = z\}$. Show that $\text{ADD} \in L$.
 - (b) Let $\text{PALADD} = \{\langle x, y \rangle : x, y > 0 \text{ are binary integers where } x + y \text{ is an integer whose binary representation is a palindrome}\}$. (Note that the binary representation of the sum is assumed not to have leading zeros. A palindrome is a string that equals its reverse.) Show that $\text{PALADD} \in L$.
6. (Sipser #8.27) Recall that a directed graph is strongly connected if every two nodes are connected by a directed path in each direction. Let $\text{STRONGLYCONNECTED} = \{\langle G \rangle : G \text{ is a strongly connected graph}\}$. Show that STRONGLYCONNECTED is NL-complete.