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| **cs579 Computational Complexity** | Out: *Thu., 2024-09-12* |
| Problem Set #2 | |
| Prof. Michael A. Forbes | Due: *Thu., 2024-09-26 12:00* |

All problems are of equal value.

1. (Sipser #7.21) Let $\mathsf{DOUBLESAT} = \{\langle\varphi\rangle : \varphi$ has at least two satisfying assignments$\}$. Show that $\mathsf{DOUBLESAT}$ is $\mathsf{NP}$-complete.

2. (Sipser #7.39) In the proof of the Cook-Levin theorem, a window is a $2 \times 3$ rectangle of cells. Show why the proof would have failed if we had used $2 \times 2$ windows instead.

3. (Sipser #7.17) Show that, if $\mathsf{P} = \mathsf{NP}$, then every language $A \in \mathsf{P}$, except $A = \emptyset$ and $A = \Sigma^*$, is $\mathsf{NP}$-complete.

4. (Sipser #8.12) Show that $\mathsf{TQBF}$ restricted to formulas where the part following the quantifiers is in conjunctive normal form is still $\mathsf{PSPACE}$-complete.

5. (Sipser #8.22)
   (a) Let $\mathsf{ADD} = \{\langle x, y, z\rangle : x, y, z > 0$ are binary integers and $x + y = z\}$. Show that $\mathsf{ADD} \in \mathsf{L}$.
   (b) Let $\mathsf{PALADD} = \{\langle x, y\rangle : x, y > 0$ are binary integers where $x + y$ is an integer whose binary representation is a palindrome$\}$. (Note that the binary representation of the sum is assumed not to have leading zeros. A palindrome is a string that equals its reverse.) Show that $\mathsf{PALADD} \in \mathsf{L}$.

6. (Sipser #8.27) Recall that a directed graph is strongly connected if every two nodes are connected by a directed path in each direction. Let $\mathsf{STRONGLYCONNECTED} = \{\langle G\rangle : G$ is a strongly connected graph$\}$. Show that $\mathsf{STRONGLYCONNECTED}$ is $\mathsf{NL}$-complete.