cs579 Computational Complexity

Out: Thu., 2023-09-07

Problem Set #2

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Due: Thu., 2023-09-21 12:00

All problems are of equal value.

- 1. (Sipser #7.21) Let $\mathsf{DOUBLESAT} = \{\langle \varphi \rangle : \varphi \text{ has at least two satisfying assignments}\}$. Show that $\mathsf{DOUBLESAT}$ is NP-complete.
- 2. (Sipser #7.39) In the proof of the Cook-Levin theorem, a window is a 2×3 rectangle of cells. Show why the proof would have failed if we had used 2×2 windows instead.
- 3. (Sipser #7.17) Show that, if P = NP, then every language $A \in P$, except $A = \emptyset$ and $A = \Sigma^*$, is NP-complete.
- 4. (Sipser #8.12) Show that TQBF restricted to formulas where the part following the quantifiers is in conjunctive normal form is still PSPACE-complete.
- 5. (Sipser #8.22)
 - (a) Let $\mathsf{ADD} = \{ \langle x, y, z \rangle : x, y, z > 0 \text{ are binary integers and } x + y = z \}$. Show that $\mathsf{ADD} \in \mathsf{L}$.
 - (b) Let $\mathsf{PALADD} = \{\langle x, y \rangle : x, y > 0 \text{ are binary integers where } x + y \text{ is an integer whose binary representation is a palindrome}\}$. (Note that the binary representation of the sum is assumed not to have leading zeros. A palindrome is a string that equals its reverse.) Show that $\mathsf{PALADD} \in \mathsf{L}$.
- 6. (Sipser #8.27) Recall that a directed graph is strongly connected if every two nodes are connected by a directed path in each direction. Let STRONGLYCONNECTED = { $\langle G \rangle : G$ is a strongly connected graph}. Show that STRONGLYCONNECTED is NL-complete.