CS576 Topics in Automated Deduction

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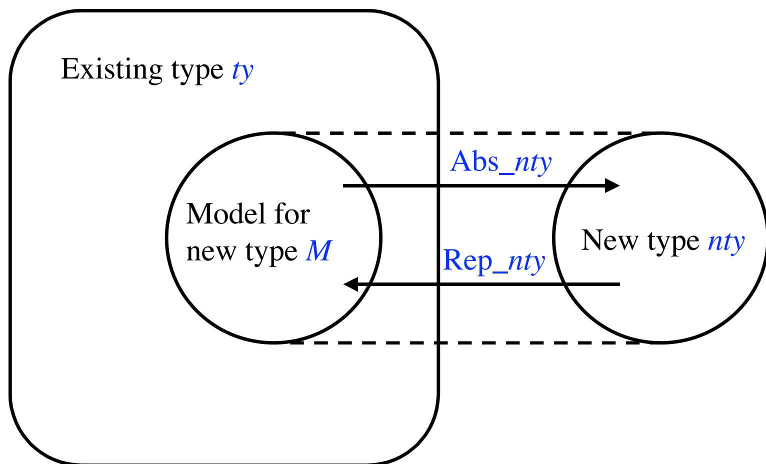
Introducing new types



Properties of a Defined Type Syntax for `typedef`:

$$\texttt{typedef}\ nty = \text{"}\mathit{modeling_set}\text{"}$$
$$\mathit{Proof\ of}\ \exists x.x \in \mathit{modeling_set}.$$

introduces a new type named *nty*, and functions

$$\texttt{Abs}_nty :: ty \Rightarrow new_ty$$
$$\texttt{Rep}_nty :: new_ty \Rightarrow ty$$

where $\mathit{modeling_set}::ty$

Properties of a Defined Type Theorems automatically provided:

Rep_*nty*: $\texttt{Rep_}nty\ x \in modeling_set$

Abs_*nty*_inverse: $\texttt{Abs_}nty(\texttt{Rep_}nty\ x) = x$

Rep_*nty*_inverse:

$y \in modeling_set \Longrightarrow \texttt{Rep_}nty(\texttt{Abs_}nty\ y) = y$

Abs_*nty*_inject:

$[|x \in modeling_set : y \in modeling_set|] \Longrightarrow$
$(\texttt{Abs_}nty\ x = \texttt{Abs_}nty\ y) = (x = y)$

Rep_*nty*_inject: $(\texttt{Rep_}nty\ x = \texttt{Rep_}nty\ y) = (x = y)$

Records in Isabelle/HOL

- Records in HOL are basically tuples, but...

```
record ('a)graph_sig =
Vertices :: "'a set"
Edges :: "('a × 'a) set"

definition one :: "(unit) graph_sig" where
"one ≡ ⦇ Vertices = {()}, Edges = {((),())} ⦈"
```

- Components accessed by field

```
lemma "()∈ Vertices one"
by (simp add: one_def)
```

- Unlike functional programming languages, position of fields matters

```
⦇ Edges = {((),())}, Vertices = {()} ⦈"
```

causes an "Error in record input"

Record Field Update

- Records support field update

```
definition no_edge :: "(unit) graph_sig" where
"no_edge = one⦇ Edges := {} ⦈"

lemma "no_edge = ⦇ Vertices = {()}, Edges = {} ⦈"
by (simp add: no_edge_def one_def)
```

Record Field Update

- Record update is functional

```
definition no_edge :: "(unit) graph_sig" where
"no_edge = one⦇Edges := {}⦈"

lemma "no_edge = ⦇Vertices = {()}, Edges = {} ⦈"
by (simp add: no_edge_def one_def)

lemma "no_edge ≠ one"
by (simp add: no_edge_def one_def)

lemma "(Vertices no_edge = Vertices one) ∧
       (Edges no_edge = {})"
by (simp add: no_edge_def one_def)
```

record type representations

- Every record type many be given by field syntax:

```
lemma "(one::unit graph_sig) =
      (one::⦇Vertices::unit set, Edges::(unit × unit) set⦈)"
by (rule refl)
```

- Every record is extensible
  - Every record `rec_ty` defines a polymorphic type `('a) rec_ty_scheme`;
    type `rec_ty` same as `(unit) rec_ty_scheme`

```
term "one::(unit,unit) graph_sig_scheme"
```

- Every record type has a "hidden field" `more`

New Record Types From Old

- New record types may be created by extending existing record types with addtional fields:

```
record ('a,'b) labeled_graph_sig = "'a graph_sig" +
Label ::  "('a × 'a) ⇒ ('b) option"

definition two ::  "⦇Vertices ::  nat set,
                 Edges ::  (nat × nat) set,
                 Label ::  nat × nat ⇒ bool option⦈" where
"two ≡
 ⦇ Vertices = {1,2}, Edges = {(1,2)},
   Label = (λ (m,n).  (if (m,n) = (1,2) then Some True else No

constants
 two ::  "(nat, bool) labeled_graph_sig"
```

Record Polymorphism

- The _scheme types allow for (weak) record polymorphism
- Also referrd to as record subtyping
- Generally, input to functions should use _scheme type instead of strict record type

```
definition is_graph :: "('a,'b) graph_sig_scheme ⇒ bool" wher
"is_graph G ≡ (∀ e ∈ Edges G. {fst e, snd e} ⊆ Vertices G)"

lemma shows "is_graph one"
by (simp add: one_def is_graph_def)

lemma shows "is_graph two"
by (simp add: two_def is_graph_def)
```