4/6/21
- min cut
- matchings

| w_1 | w_2 | w_3 | ... w_n |

$w_i \in \mathbb{N}$, $\forall\ w_i > 0$

random (k) returns integer O(1) uniformly in 1...k

Q: How pick one of the w_i s.t. $P[X = i] = \frac{w_i}{\sum_\ell w_\ell}$

O(n)

| w_1 | w_2 | w_3 | ... |

0 | 1 | 2 | 3 | 4 | 5 ... |

$w_1 = 3$, w_2, w_3

$I(w_i) = \left[\sum_{\ell<i} w_\ell + 1, \sum_{\ell \le i} w_i \right]$

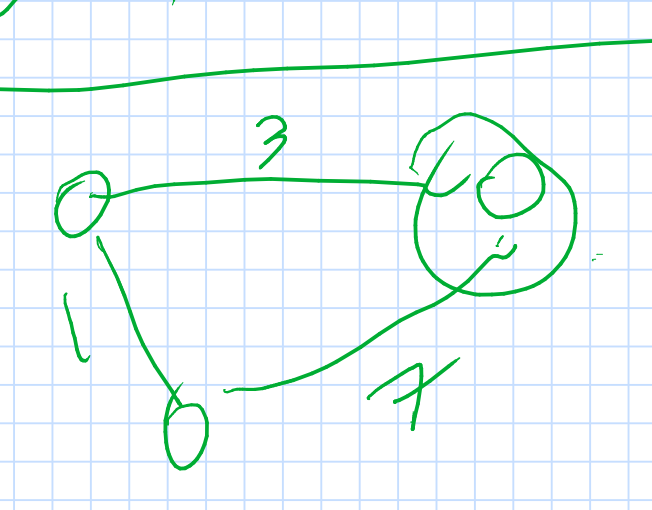
w_1
$w_1 + w_2$
$w_1 + w_2 + w_3$

prefix sums

| w_1 | w_2 | w_3 | w_4 | |

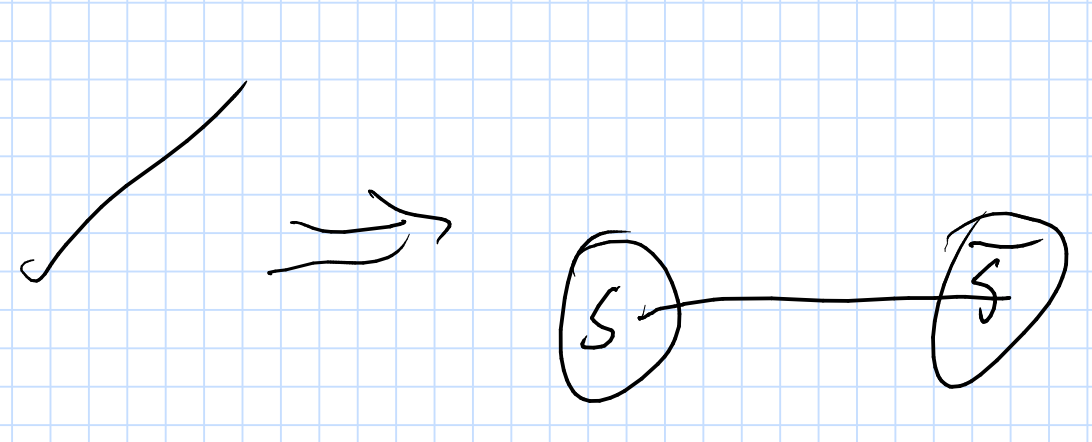
prefix sums

O(n)



O(1)

G n



$O(n^2)$

$\ge \frac{2}{n^2}$

$n^2 \log n$

$O(n^4 \log n)$

Amplification

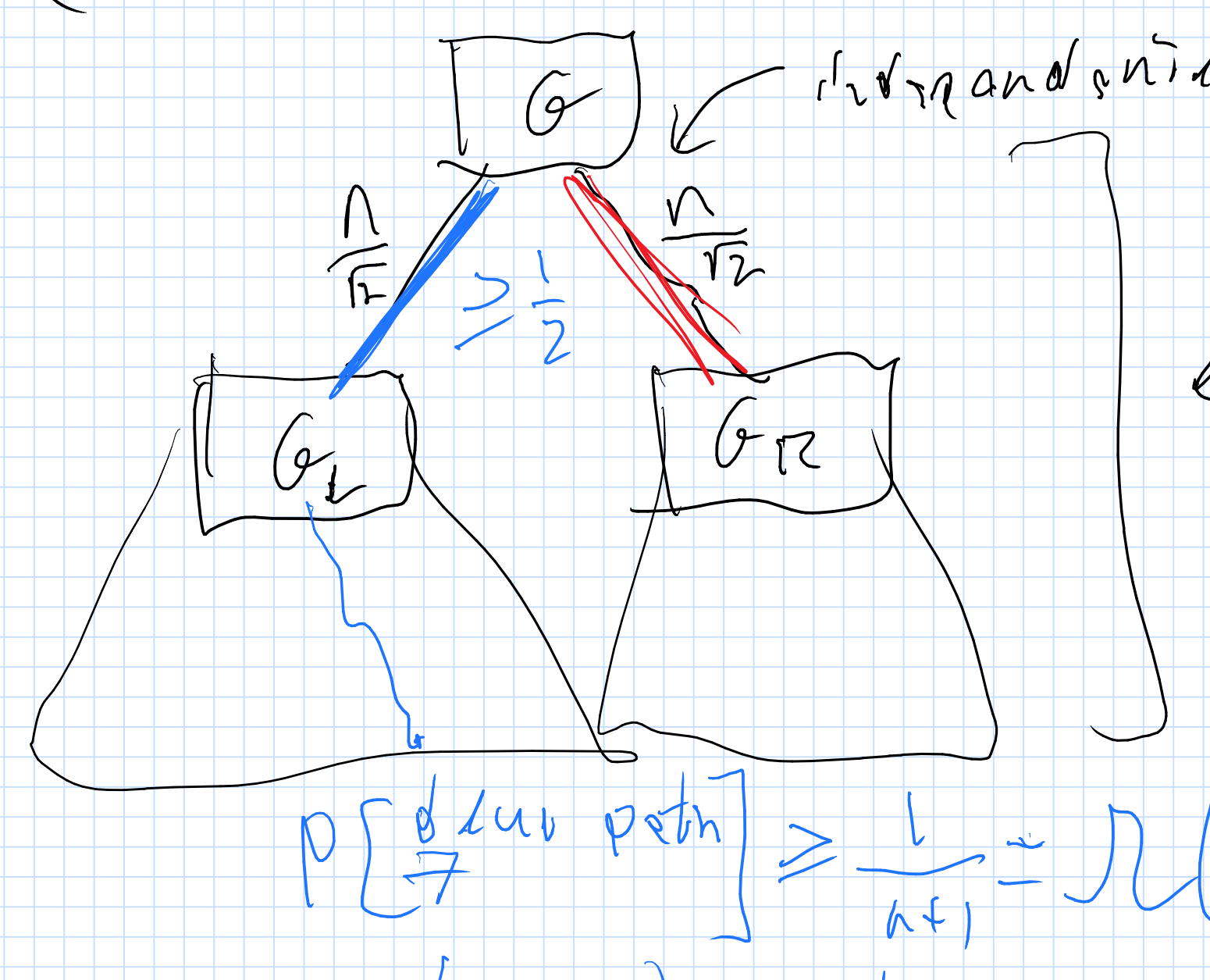
run $O\left(\frac{\log n}{p}\right)$ basic alg

$\frac{n-2}{n} \cdot \frac{n-3}{n-1} \cdots \rightarrow \frac{1}{2} \approx \frac{1}{n^2}$

$\frac{n-2}{n} \cdot \frac{n-3}{n-1} \cdots \frac{n/\sqrt{2}}{n/\sqrt{2}+2}$

$\approx \frac{(n/\sqrt{2}+1)(n/\sqrt{2})}{n(n-1)} \ge \frac{n^2}{\sqrt{2}\sqrt{2}n^2} = \frac{1}{2}$

$\left(n - \frac{n}{\sqrt{2}}\right) O(n) = O(n^2)$



$P[\exists \text{ dual path}] \ge \frac{1}{n+1} = \Omega\left(\frac{1}{\log n}\right)$

$O(\log n)$: # times we need to run alg

$T(n) = O(n^2) + 2T\left(\frac{n}{\sqrt{2}}\right)$

$T(n) = O(n^2) + 2\left(O\left(\left(\frac{n}{\sqrt{2}}\right)^2\right) + 2T\left(\frac{n}{2}\right)\right)$

$= O(n^2) + O(n^2) + 4T\left(\frac{n}{2}\right)$

$O(n^2 \log n)$ ←

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basic	fast	
$O(n^2)$	$O(n^2 \log n)$	
$\frac{1}{n^2}$	$\frac{1}{2\log_2 n}$	
$O(n^2 \log n)$	$O(\log^2 n)$	# time to run
$O(n^4 \log n)$	$O(n^2 \log^3 n)$	

David Karger

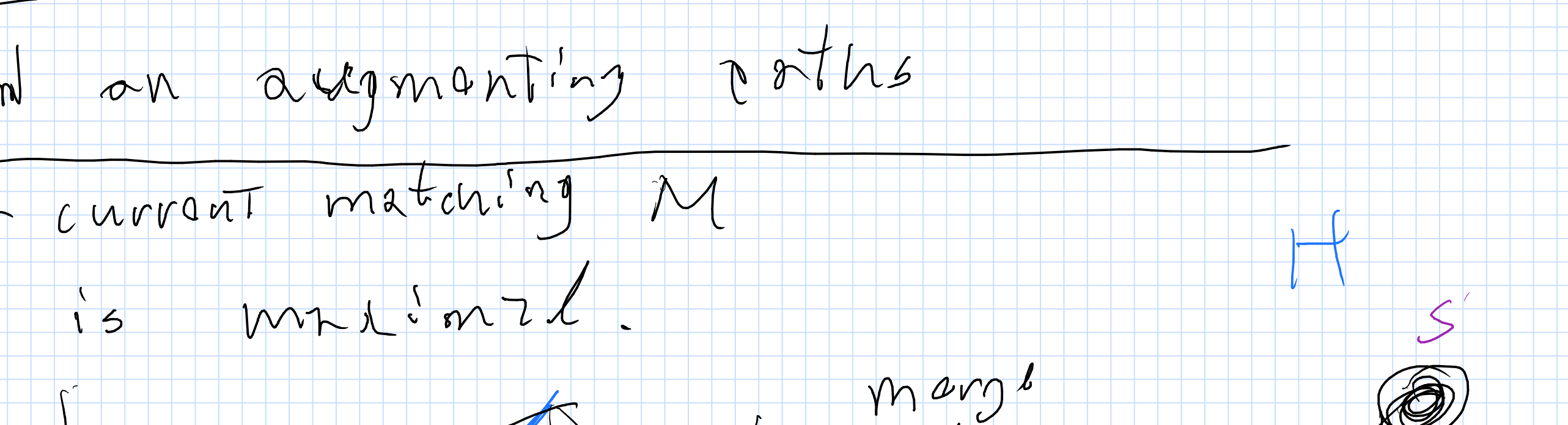
Matchings

General graphs — Unweighted case

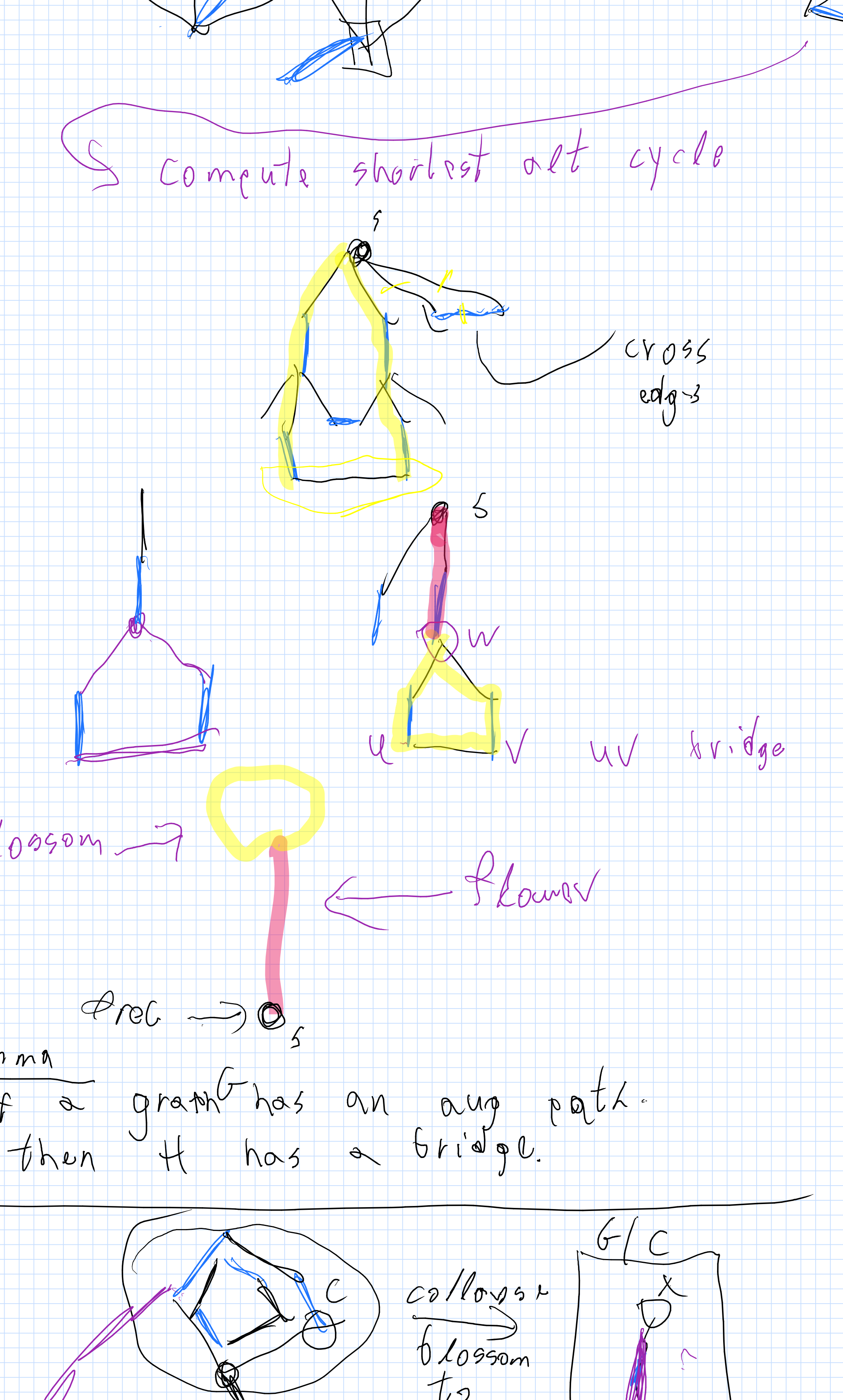
Find an augmenting paths

G - current matching M

M is maximal.

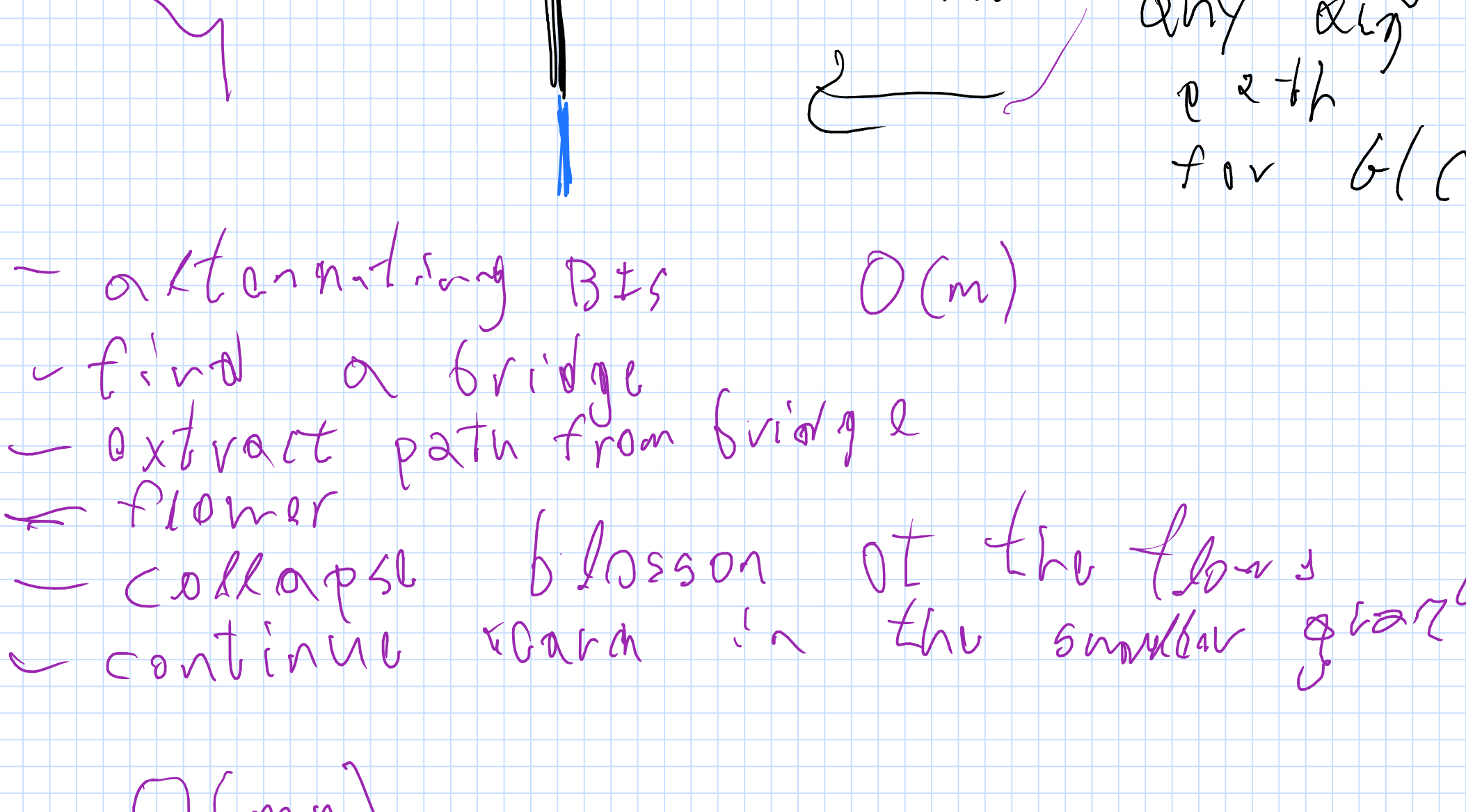


compute shortest alt cycle



Lemma

If a graph G has an aug path then H has a bridge.



- alternating BFS O(m)
- find a bridge
- extract path from bridge
- flower
- collapse blossom of the flower
- continue search in the smaller graph

O(mn)

$O(mn^2)$ overall running time.

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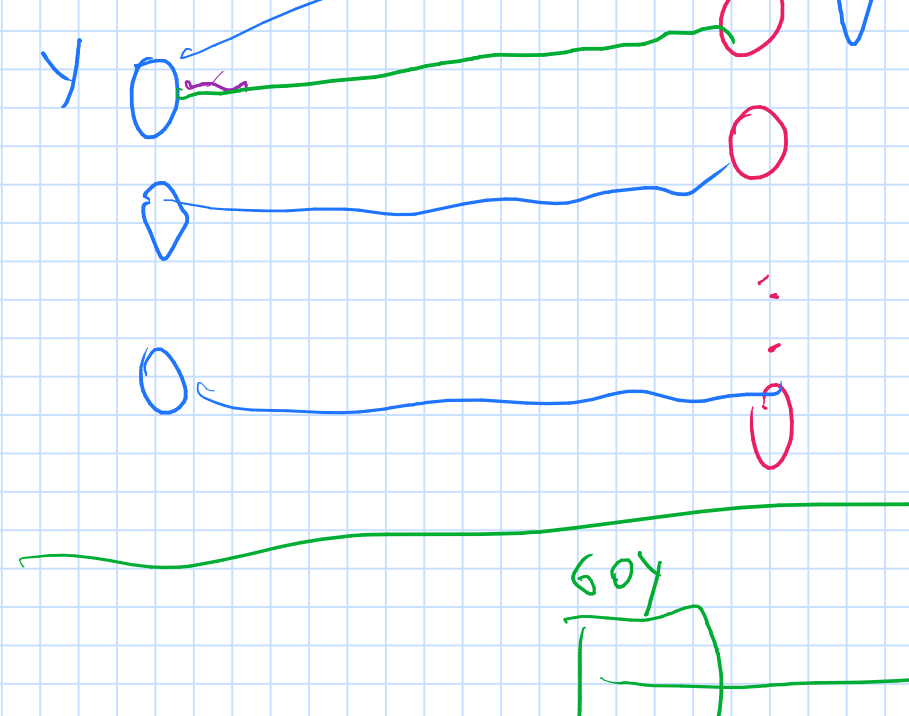
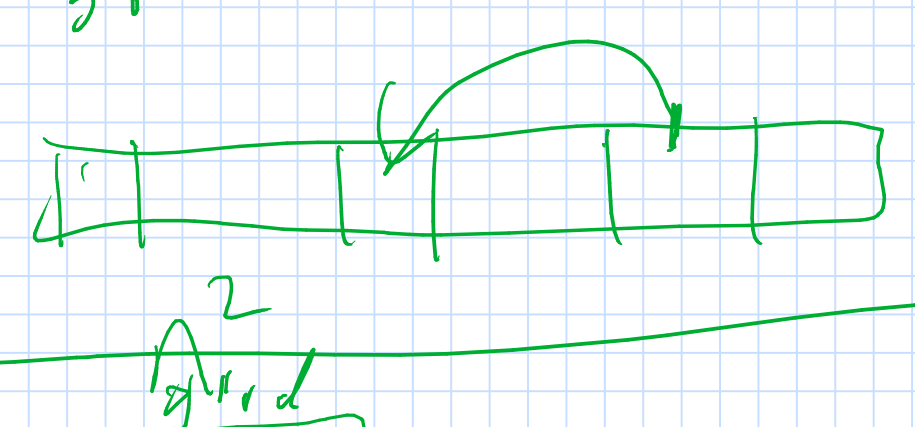
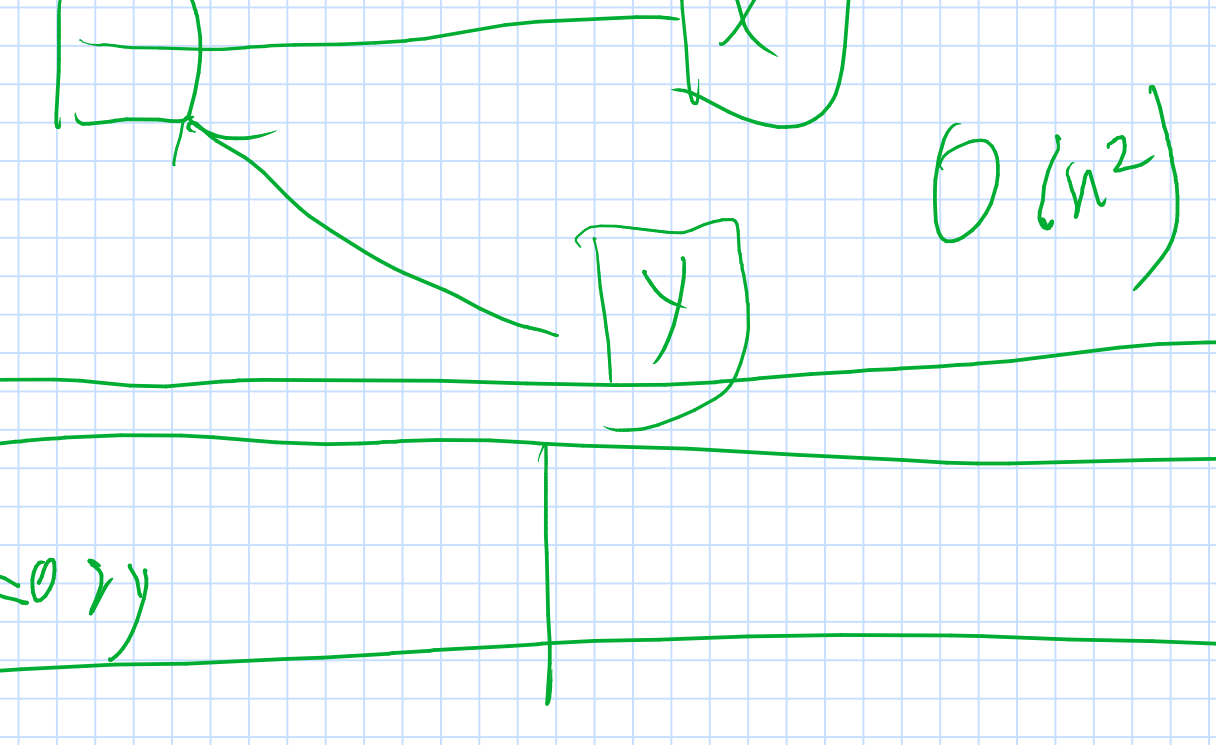
stable matchings (Medical schools in the USA)



stable matching





$O(n^2)$

Game theory

EC