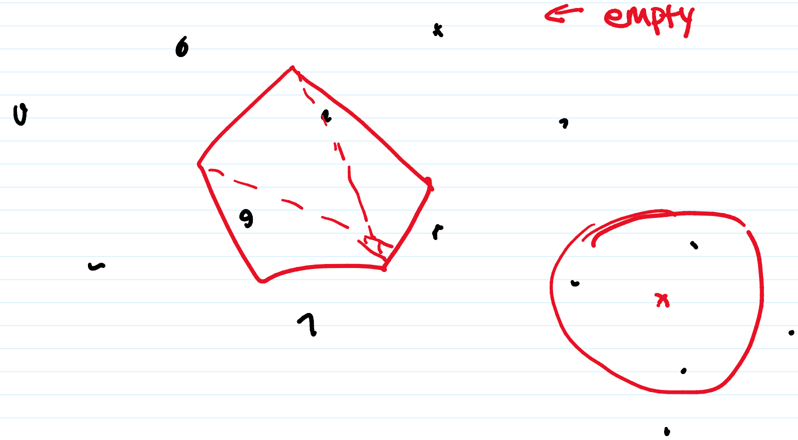
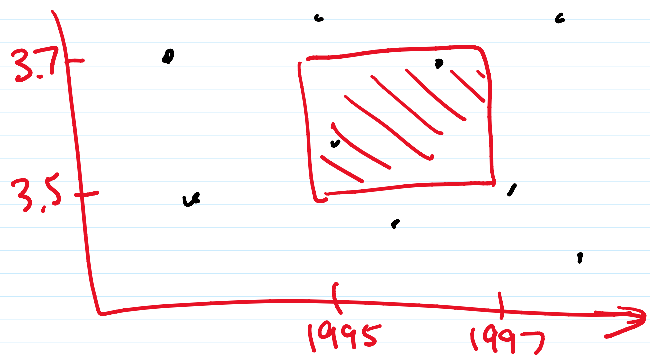
November 21, 2024 3:18 PM

Range Searching

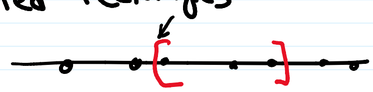
Problem preprocess a set P of n pts in $\mathbb{R}^d$
s.t. given a query region R,
can find pts in $P \cap R$ ← report all or ← count ← empty





Orthogonal case

ranges R are axis-aligned rectangles



1D: $P(n) = O(n \log n)$
$S(n) = O(n)$
$Q(n) = O(\log n)$ emptiness/count
$O(\log n + k)$ report
(k = output size)

$(U(n) = O(\log n)$ update for dynamic$)$

Method 1: k-d Tree

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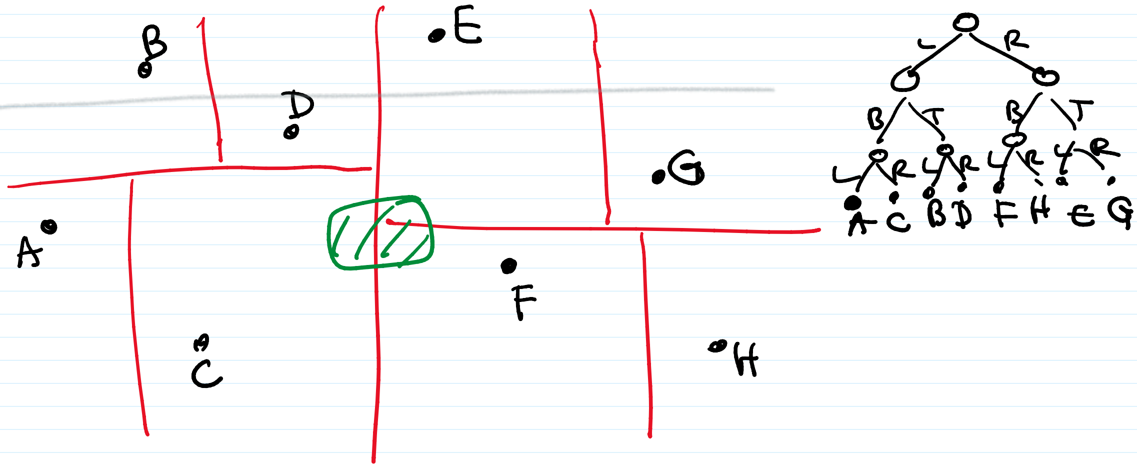


alternately split by median-x, then median-y

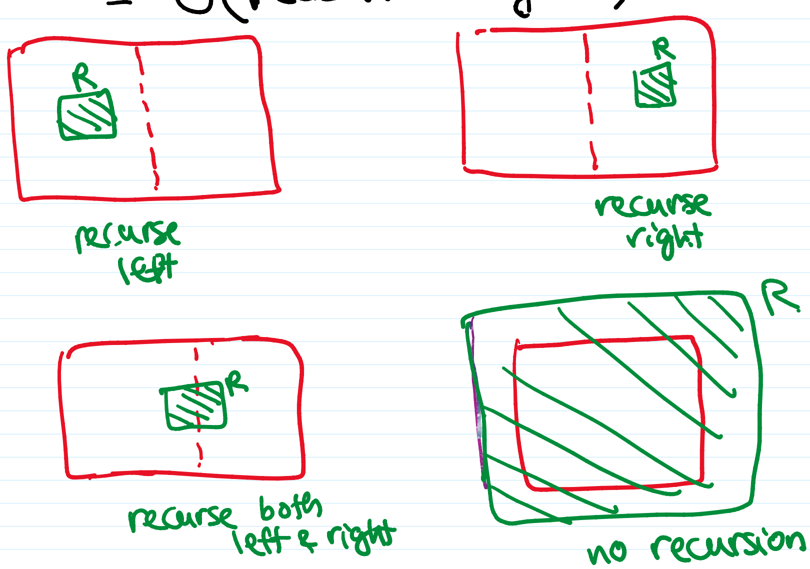
$$P(n) = 2P(n/2) + O(n) \Rightarrow P(n) = \boxed{O(n\log n)}$$

$$S(n) = \boxed{O(n)}$$

$$Q(n) = \cancel{2Q(n/2) + O(1) \Rightarrow O(n)}$$

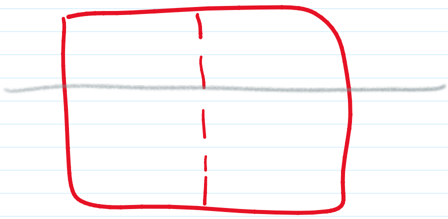
$$Q(n) = O(\#\text{cells visited})$$

$$= O(\#\text{cells intersecting } \partial R) \cdot \log n?$$

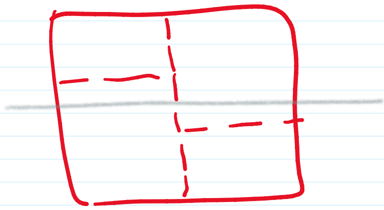


$$= O(\cancel{4\times}\ \#\text{cells intersecting a vertical/horizontal line } \cancel{\text{seg}})$$

let $f(n)$ = max # cells intersecting a vertical/horizontal line



$f(n) = \cancel{2f(n/2) + 1}$

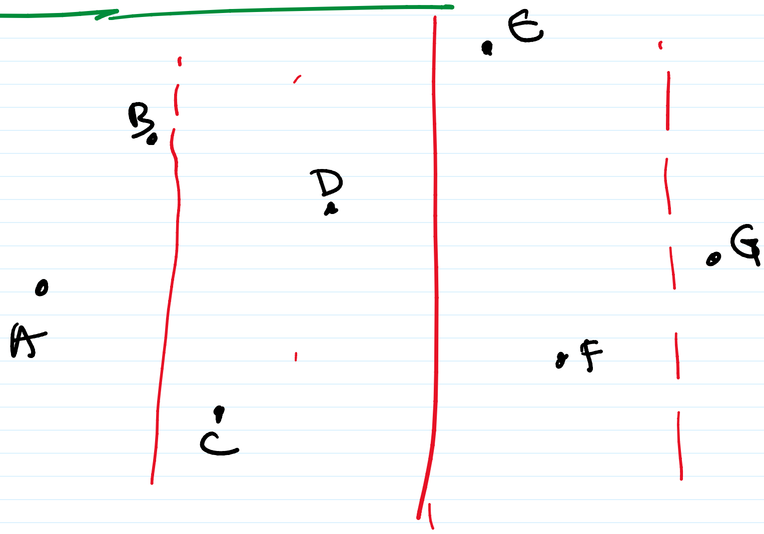


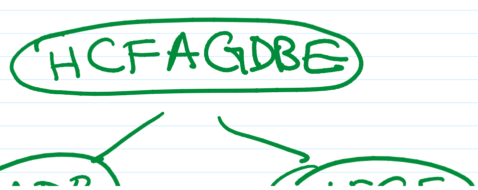
$$f(n) = 2f\left(\frac{n}{4}\right) + O(1)$$
$$\Rightarrow O\left(n^{\log_4 2}\right)$$
$$= O(\sqrt{n})$$

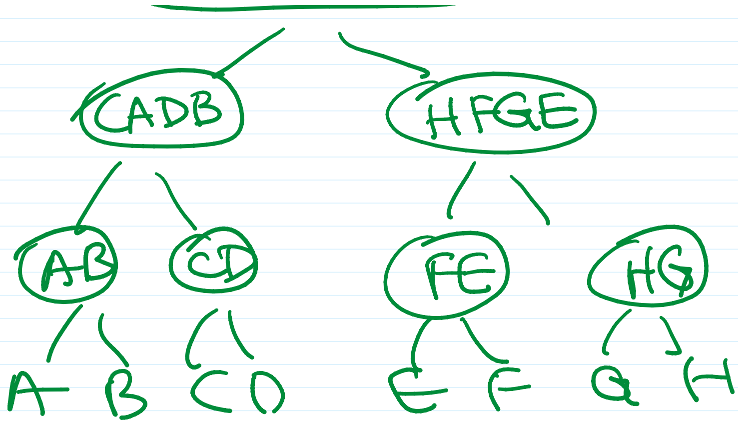
$$Q(n) = \boxed{O(\sqrt{n})} \text{ (+ k for reporting)}$$

(in higher-d, $f(n) = 2^{d-1} f\left(\frac{n}{2^d}\right) + O(1)$
$\Rightarrow O\left(n^{\frac{d-1}{d}}\right) = O\left(n^{1-\frac{1}{d}}\right)$)

Method 2: Range Tree







$\text{preprocess}_{\underset{d}{2}}(P)$:

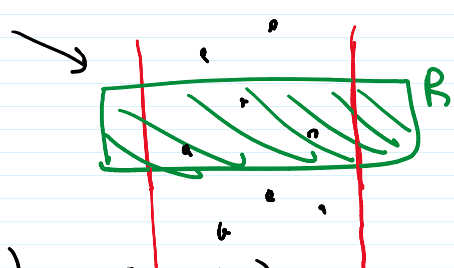
$x_m = \text{median-}x$ ← just sorting
$\text{preprocess}_{\underset{d-1}{1}}(\{p.y : p \in P\})$
left → $\text{preprocess}_{\underset{d}{2}}(\{p \in P : p.x \le x_m\})$
right → $\text{preprocess}_{\underset{d}{2}}(\{p \in P : p.x > x_m\})$

$S(n) = \boxed{O(n \log n)}$ (each level is $O(n)$)
$\left(S(n) = 2S(\frac{n}{2}) + O(n) \right)$

$P(n) = 2P(\frac{n}{2}) + O(n \log n) \Rightarrow \boxed{O(n \log^2 n)}$
(by pre-sort)

$\text{query}_2(R)$:
if R doesn't intersect slab return $\emptyset$
if R is long in slab return query_1(proj of R by y).

left → $\text{query}_2(R)$
right → $\text{query}_2(R)$



$Q_2(n) = O\left((\text{\# slabs R is short}) * Q_1(n) \right)$

$$Q_2(n) = O(\underbrace{(\#\text{ slabs } R \text{ is short})}_{2\log n} * \underbrace{Q_1(n)}_{\log n})$$

$$= \boxed{O(\log^2 n)} \quad (+k \text{ for reporting})$$

$O(\log n)$ by using more ptrs

higher d:

$$S_d(n) = \boxed{O(n \log^{d-1} n)}$$

$$Q_d(n) = \boxed{O(\log^{d-1} n)}$$