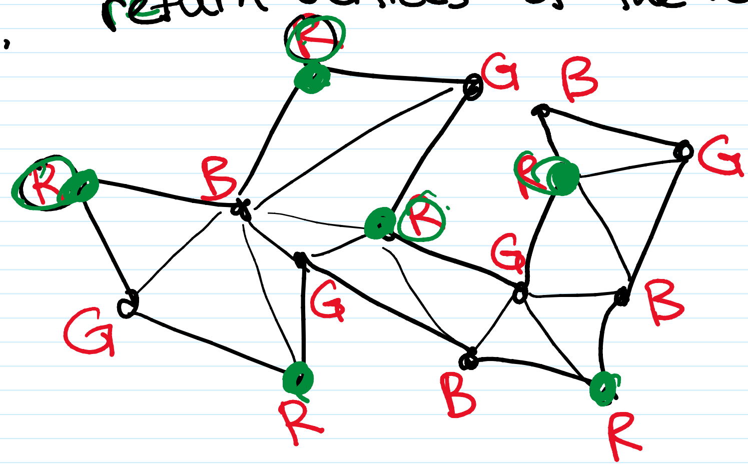
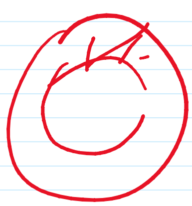
\# guards $= \lfloor n/3 \rfloor$ for this polygon

Chvatal's Thm ('75) for every simple polygon,
\# guards $\leq \lfloor n/3 \rfloor$.

Fisk's Pf ('78):

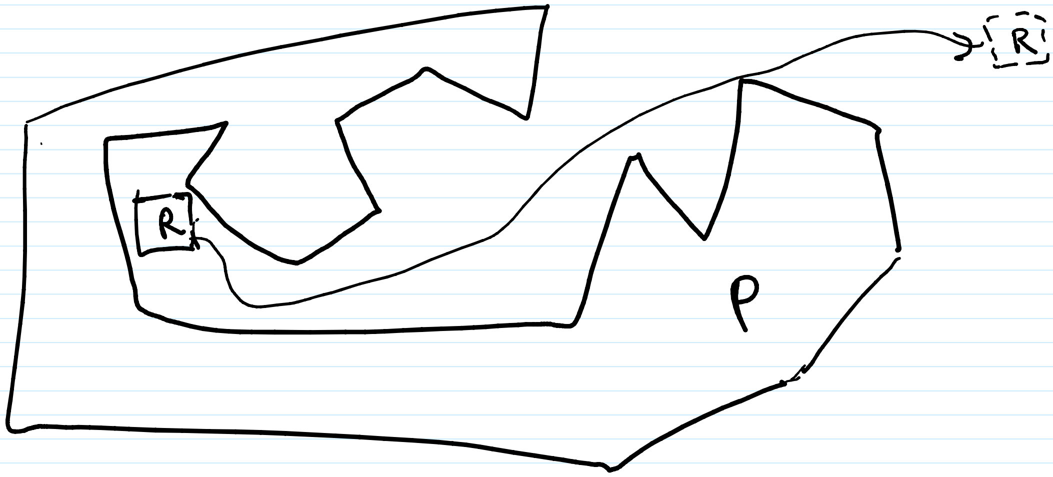
1. compute a triangulation T
2. compute a 3-coloring of T
3. return vertices of the least popular color $\leq \lfloor \frac{n}{3} \rfloor$.





A Motion Planning Application

Given a robot R (convex polygon of const complexity)
& an environment P (simple polygon of n vertices),



move R from position t_0 to t_1,
avoiding P.

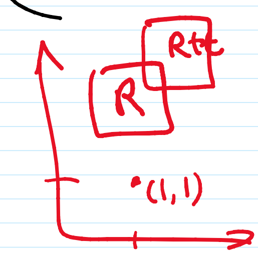
here, translate only

Define forbidden space

$$F = \{ t \in \mathbb{R}^2 : (R+t) \cap P \neq \emptyset \}$$

$$= \{ t \in \mathbb{R}^2 : \exists\, r \in R,\ p \in P,\ \underbrace{r+t=p}_{t=p-r} \}$$

$(R+t = \{ r+t : r \in R \})$



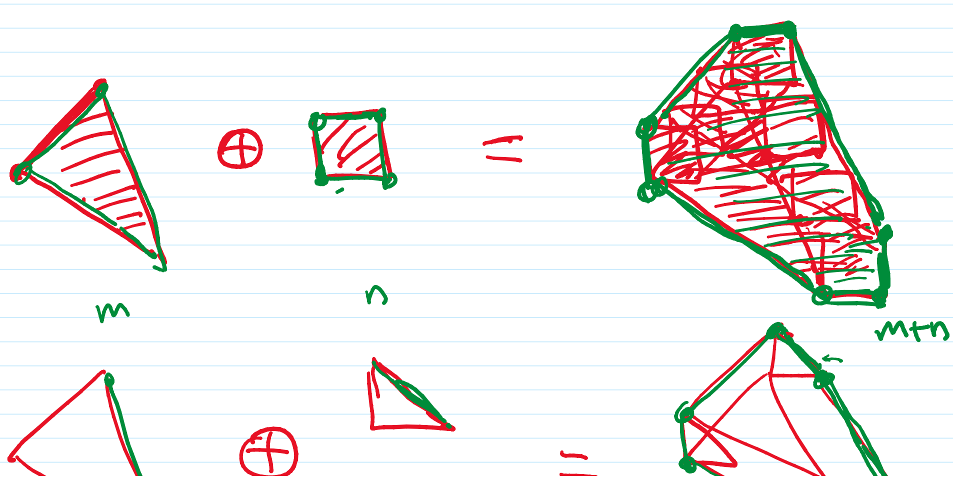
$$= \{ p-r : p \in P,\ r \in R \}$$

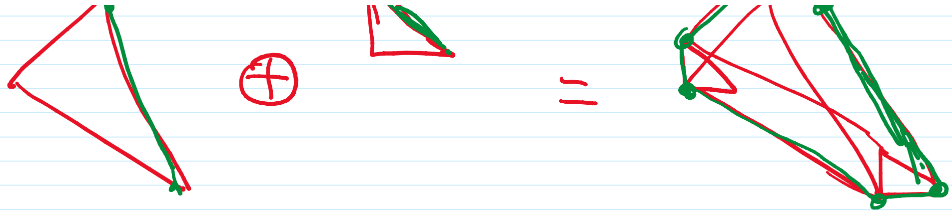
$$= \{ p+q : p \in P,\ q \in Q \} \qquad \text{let } Q = -R$$

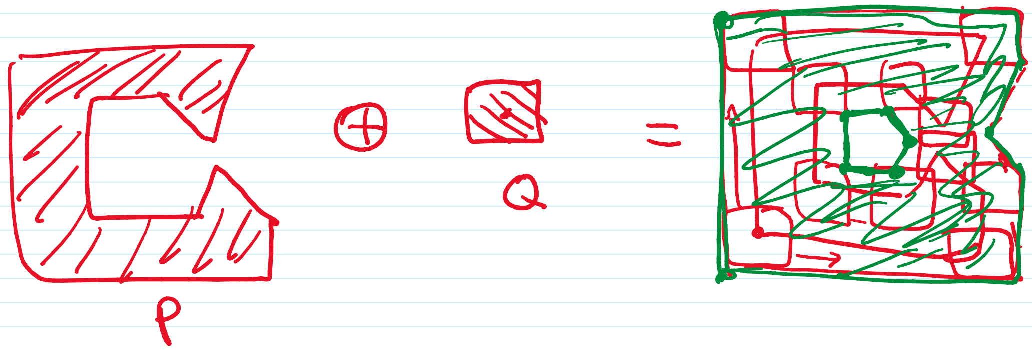
$$= P \oplus Q$$

called Minkowski sum

e.g.







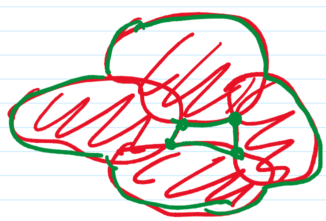
Algm:

1. triangulate $P = \bigcup_{i=1}^{n-2} \Delta_i$ (Δ_i interior-disjoint)

$O(n) \rightarrow$ 2. for each i, compute $\Delta_i' = \Delta_i \oplus Q$ (const complexity convex)

3. compute $F = P \oplus Q = \bigcup_{i=1}^{n-2} \Delta_i'$ (Δ_i' may overlap)

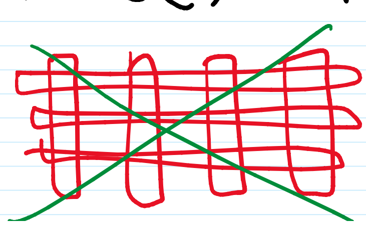
how to compute union?

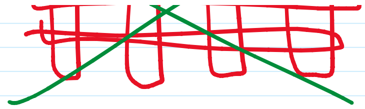


4. find path from t_0 to t_1 in $\underbrace{\mathbb{R}^2 - F}_{\text{complement}}$ "free space"

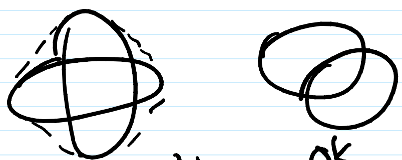
by graph search after decomposing $\mathbb{R}^2 - F$ into triangles

Obs 1 union has $O(n)$ complexity ← # vertices/edges



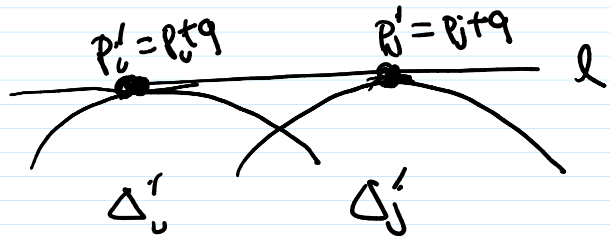


Pf: Each pair Δ_i', Δ_j' intersect at most twice ← called "pseudo-disks"

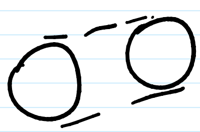


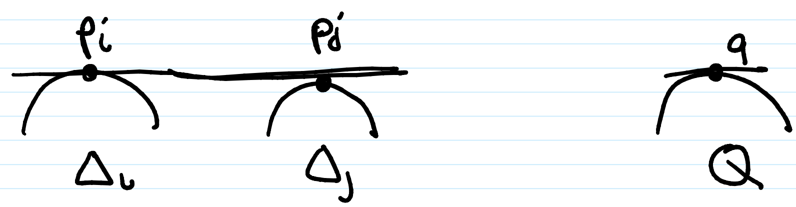
not possible ok

if not, it has > 2 common tangents



$\Delta_i' = \Delta_i \oplus Q$

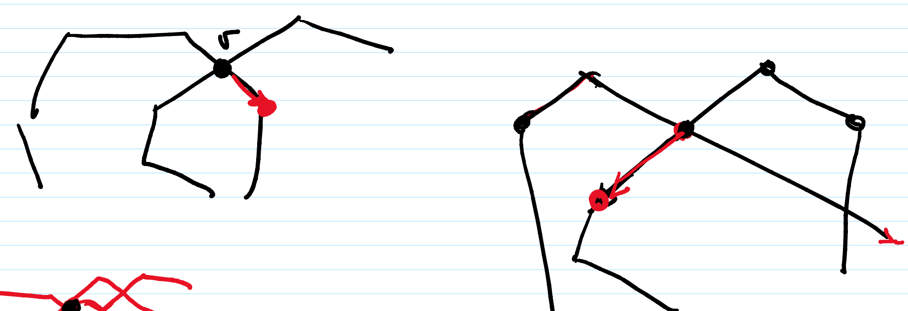


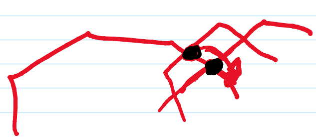


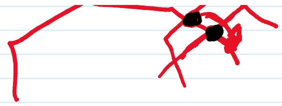
Union of n pseudo-disks has $O(n)$ complexity.

↑ convex polygons of $O(n)$ complexity

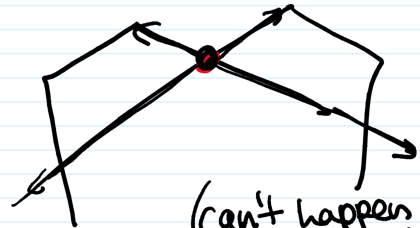
Charge each vertex v of union to a vertex of $\{\Delta_i'\}$.







each vertex of $\{\Delta_i\}$ receives ≤ 1 charge



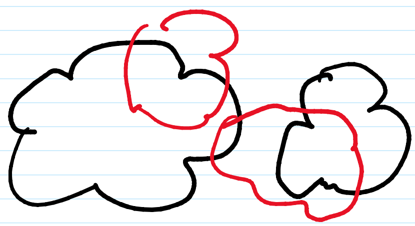
(can't happen by pseudo-disk)

Obs 2 union can be constructed in $O(n \log^2 n)$ time

Pf: divide & conquer

merging by line segment intersection alg'm with $k = O(n)$ intersect.

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n \log n)$$



$$\Rightarrow O(n \log^2 n). \quad \square$$

Total time $\boxed{O(n \log^2 n)}$.