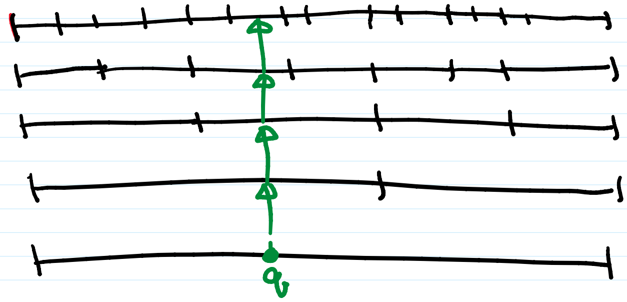
November 12, 2024 3:17 PM

POINT LOCATION

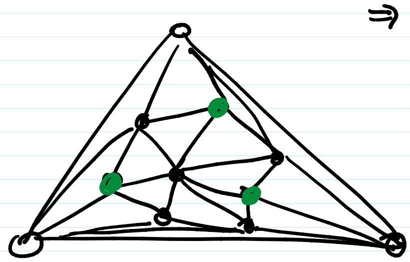
Kirkpatrick's Method: ('81)

Prune-and-Search

1D:

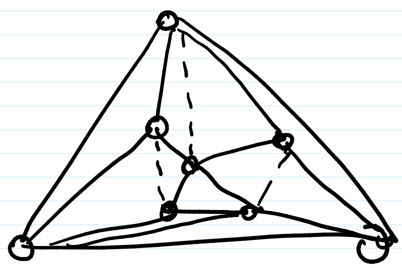


2D: assume input subdivision into triangles
⇒ triangulation



green: "independent" set
(no 2 selected vertices are adj.)

⇓

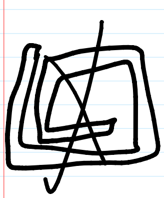


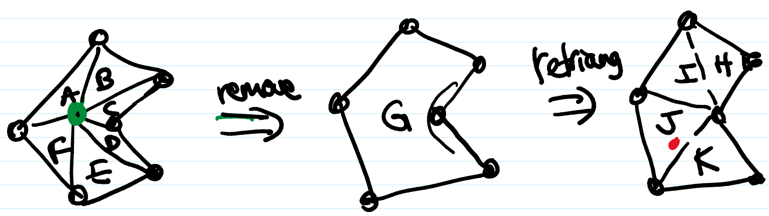
hierarchy of triangulations

⇓

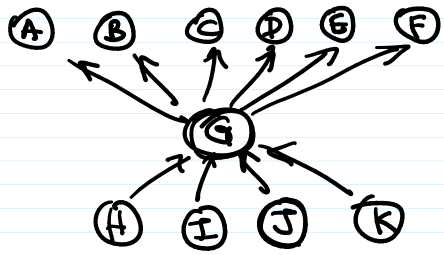
⋮

preproc(S): // given triangulation S
  repeat {
    find an indep set I of vertices
    remove I from S
    retriangulate & link old triangles to new ones
  }





(A) (B) (C) (D) (E) (F)



Query - follow links (a dag again!)

Fact Given any planar graph with n vertices, can find an indep set I of size $\geq \frac{n}{24}$ vertices where each $v \in I$ has deg ≤ 11. in $\underline{O(n)}$ time.

Pf: Greedy alg'm:

$O(n)$
1. mark all vertices of deg ≥ 12
2. while some vertex v is unmarked {
3. put v in I, mark v & its neighbors
}

Correctness:

line 1 marks $\leq n/2$ vertices

[if not, $>$ $n/2$ vertices of deg ≥ 12
$\Rightarrow$ total deg $> 6n$
but total deg $= 2m < 6n$: contra!)

line 3 marks $\leq$ 12 vertices

$\Rightarrow n \leq \frac{n}{2} + 12|I|$

$\Rightarrow |I| \geq \frac{n}{24}$. $\square$

Analysis:

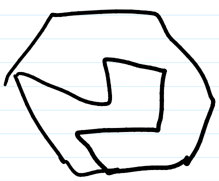
$$S(n) \leq S\left(\frac{23}{24}n\right) + O(n)$$

$$\Rightarrow S(n) = O\left(n + \frac{23}{24}n + \left(\frac{23}{24}\right)^2 n + \dots\right)$$

$$\Rightarrow \quad S(n) = O\left(n + \tfrac{23}{24}n + \left(\tfrac{23}{24}\right)^2 n + \cdots\right)$$
$$= O(24n) = \boxed{O(n)}.$$

$$Q(n) \le Q\left(\tfrac{23}{24}n\right) + O(1)$$
$$\Rightarrow \quad Q(n) = O\left(\log_{\frac{24}{23}} n\right)$$
$$= \boxed{O(\log n)}$$

$$P(n) \le P\left(\tfrac{23}{24}n\right) + O(n)$$
$$\Rightarrow \quad \boxed{O(n)}$$

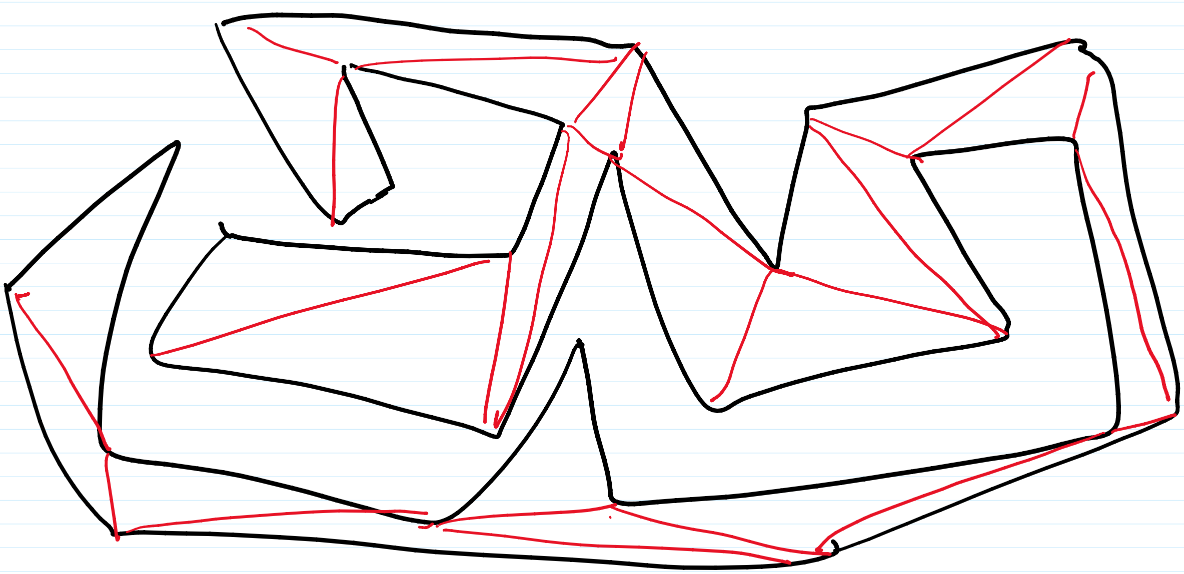


+ time to triangulate

optimal.

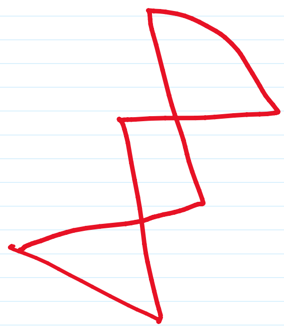
Polygon Triangulation

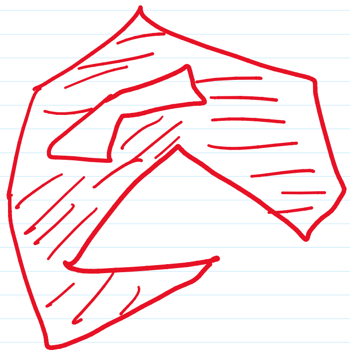
#triangles = n-2



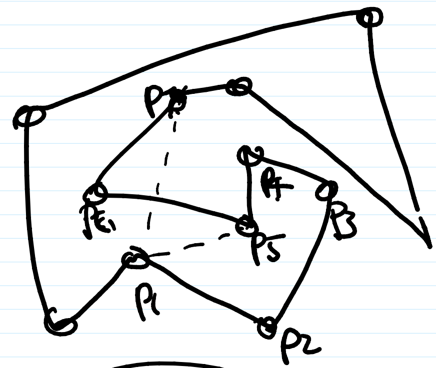
Problem Given a simple polygon P with n vertices,

Problem Given a ~~simple~~ simple polygon P with n vertices, partition P into triangles, without using new vertices.

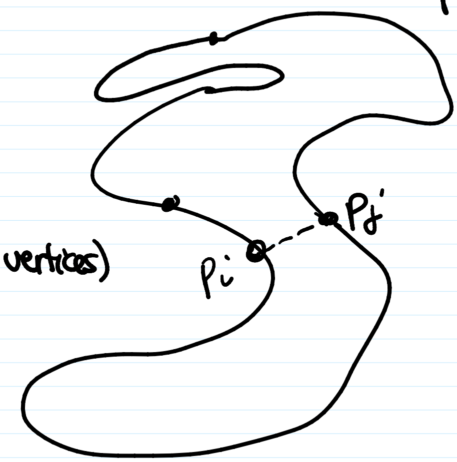




holes



p_1p_5 yes
p_1p_3 not



Earliest? Proof (Lennes 1911)

1. find a line seg p_ip_j (diagonal) not intersecting P (p_i, p_j vertices)
2. recurse on two sides

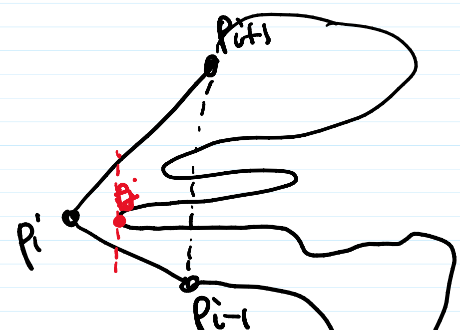
Proof that a diagonal exists:

$O(n)$ 1.1. let p_i be leftmost vertex

$O(n)$ 1.2. if $p_{i-1}p_{i+1}$ is a diagonal, return $p_{i-1}p_{i+1}$

$O(n)$ 1.3. let p_j be leftmost vertex inside $\triangle p_{i-1}p_ip_{i+1}$

1.4. return p_ip_j.



$$T(n) \le T(n_1) + T(n_2) + O(n)$$

for some $n_1 + n_2 = n + 2$

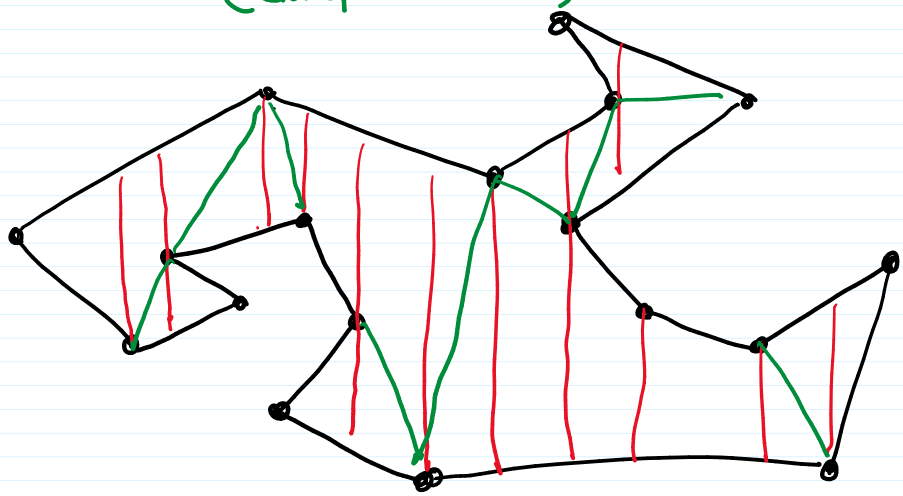
$\Rightarrow \boxed{O(n^2)}$ time

Chazelle '82: find a diagonal with $n_1, n_2 \lesssim \frac{2}{3}n$ in $O(n)$ time

$\Rightarrow O(n \log n)$

Method 2: Reduce to Trapezoidal Decomposition

(Garey et al. 1978)



1. compute TD within P in $O(n \log n)$ time (Bentley-Ottmann sweep or rand incremental)

2. for each trapezoid τ
   if τ contains 2 nonadj vertices
   draw diagonal