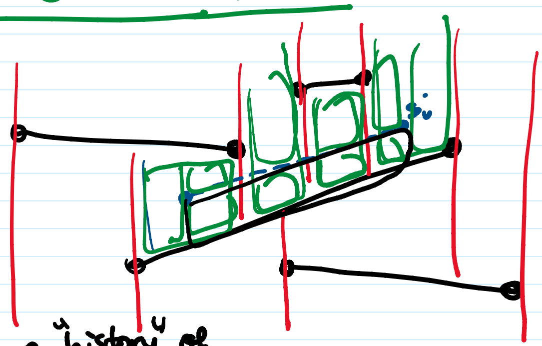
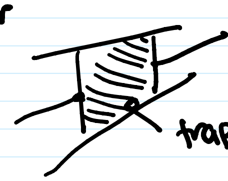
November 7, 2024 3:18 PM

Point Location

Rand. Incremental Method



maintain a "history" of the vertical/"trapezoidal decomposition" as we insert segs in rand order

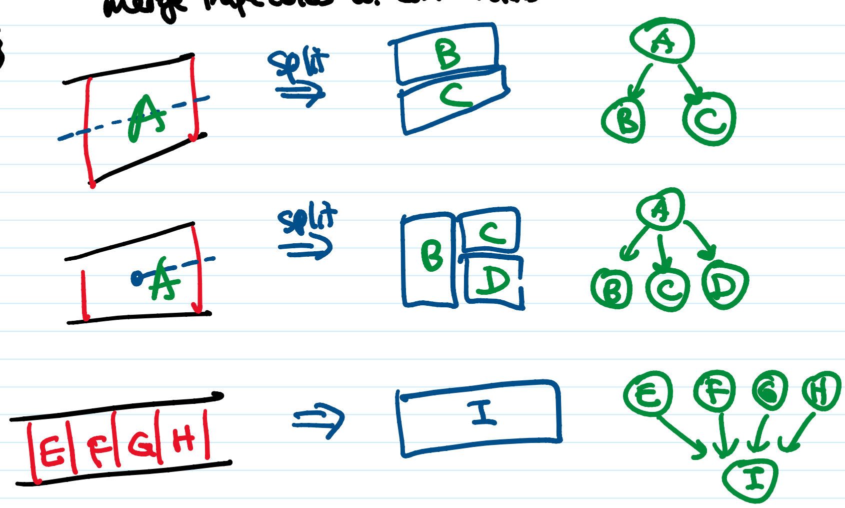


trapezoid

preproc(S):

0. let $s_1, .., s_n$ be a rand order of S
1. for $i = 1, .., n$ do {
2. locate trapezoid containing left endpt of s_i
3. generate all trapezoids cut by s_i by "walking"
4. Split these trapezoids & link old to new ones
5. merge trapezoids w. common sides " " " "

}



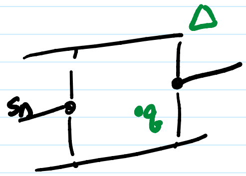
query algm - follow links from top down in history dag (directed acyclic graph)

Query Time Analysis:

assume query pt q is indep of rand. order

look at s_n (backwards analysis)

let Δ be final trapezoid containing q



$$\Pr[\text{need to follow a link created by } n^{th} \text{ iteration}]$$
$$= \Pr[s_n \text{ touches } \Delta]$$
$$\leq \frac{4}{n}.$$

expected time

$$Q(n) = \frac{4}{n} \cdot O(1) + Q(n-1)$$
$$\Rightarrow O\left(\frac{4}{n} + \frac{4}{n-1} + \frac{4}{n-2} + \ldots + \frac{4}{1}\right)$$
$$= O(\text{harmonic number } H_n)$$
$$= \boxed{O(\log n)}$$

Space Analysis:

look at n^{th} iteration

$\#$ trapezoids created $= O(\text{"degree" of } s_n)$ in final trap decomp.

total degree $\leq 4n$

$\Rightarrow$ expected $\#$ trapezoids created $= O(1)$

$\Rightarrow$ total expected space $\boxed{O(n)}$

Preprocessing:

line 2 $O(\log n)$ expected

lines 3-5 $O(n)$ total expected

$\Rightarrow \boxed{O(n \log n)}$ expected time

Rmks:

do same for 3D convex hull,
2D Delaunay triang, ...

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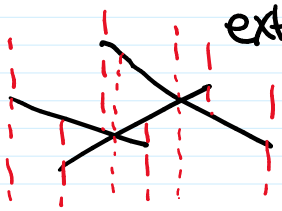
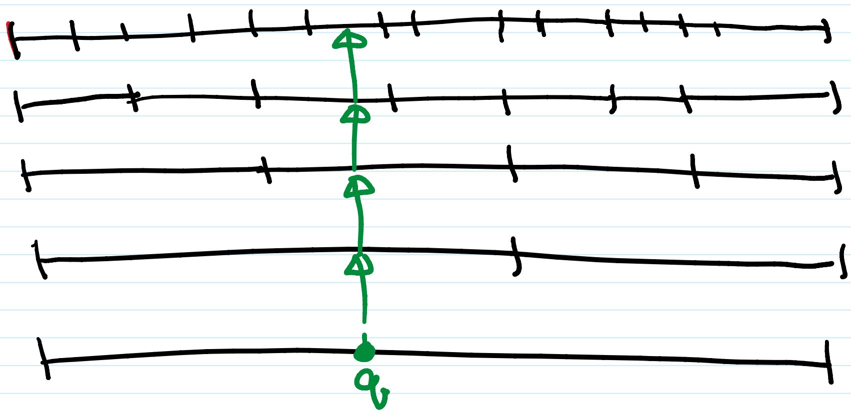


extends to intersecting segs
$O(n \log n + k)$ expected time

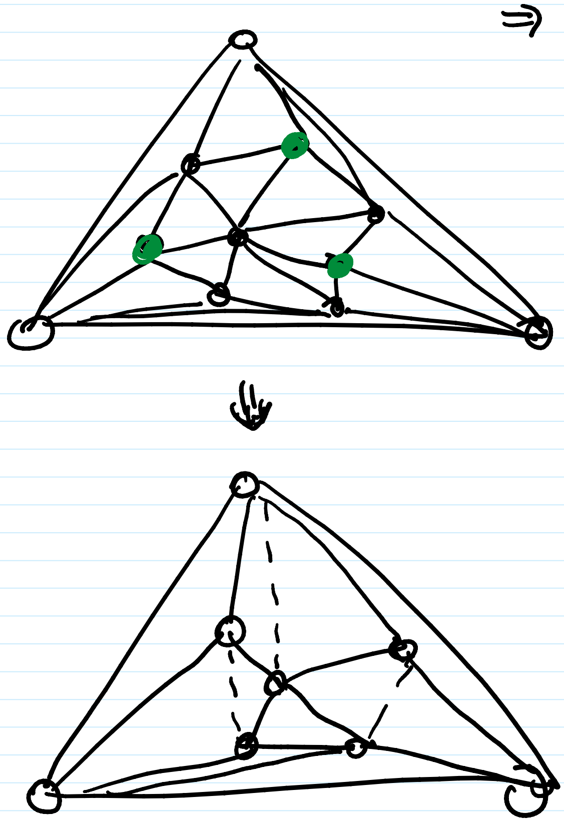
Kirkpatrick's Method: ('81)

Prune-and-Search

1D:

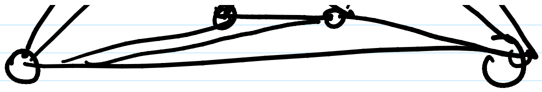


2D: assume input subdivision into triangles
$\Rightarrow$ triangulation



green: "independent" set
(no 2 selected vertices are adj.)

hierarchy of triangulations

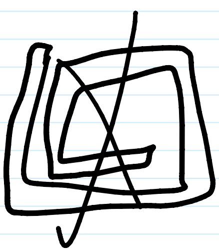


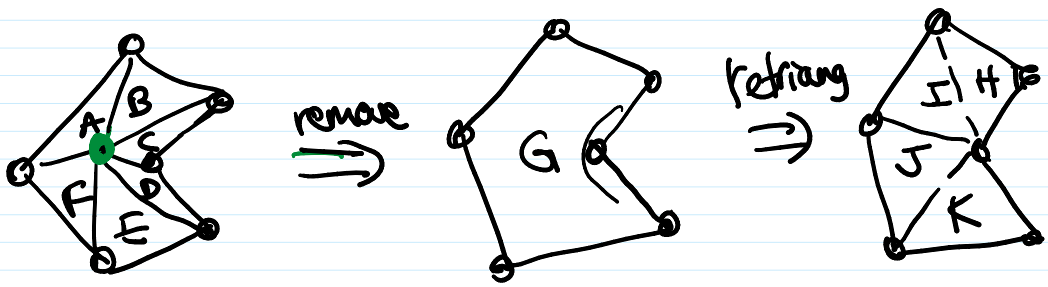
hierarchy of triangulations

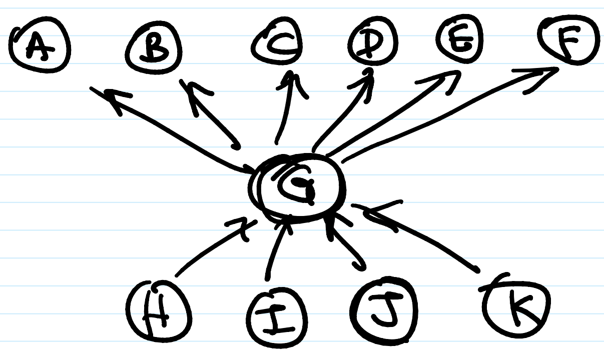
$\Downarrow$

$\vdots$

```
preproc(S):      // given triangulation S
  repeat {
    find an indep set I of vertices
    remove I from S
    retriangulate & link old triangles to new ones
  }
```







Query - follow links (a dag again!)

Fact Given any planar graph with n vertices, can find an indep set I of size $\geq \frac{n}{24}$ vertices where each $v \in I$ has $\deg \leq 11$. in $O(n)$ time.