Pf: Fix v that violates $\geq c d \frac{n}{r} \log n$ halfspaces in H.

$\Pr[$ R does not include any of violating halfspaces $]$

$$\lesssim \left(1 - \frac{c d \frac{n}{r} \log n}{n}\right)^r$$

$1-x \leq e^{-x}$

$$\lesssim \left(e^{-\frac{c d \cancel{\frac{n}{r}} \log n}{\cancel{n}}}\right)^{\cancel{r}}$$

$$\approx \frac{1}{n^{cd}}$$

$\Pr(A \cup B) \leq \Pr(A) + \Pr(B)$

$$\Pr(\text{err}) \leq n^d \cdot \frac{1}{n^{cd}} = \frac{1}{n^{(c-1)d}}.$$

$\square$

$$\Rightarrow \quad T(n) = (d+1)\, T\left(O(\sqrt{n}\, \cancel{\log n})\right) + O(d^2 n)$$

$$\Rightarrow \quad T(n) = \boxed{O(n)} \quad \text{for any const } d$$

$$O\left(d^2 n + (d \log n)^{O(d)}\right)$$

Clarkson's Second Alg'm: ("Multiplicative Weight Update")

LP(H):

repeat {

choose a random (weighted) sample $R \subseteq H$ of size $r = c d^2$

$v = LP(R)$ by brute force ←

for each $h \in H$ violated by v do

double the multiplicity (or weight) of h in H

}

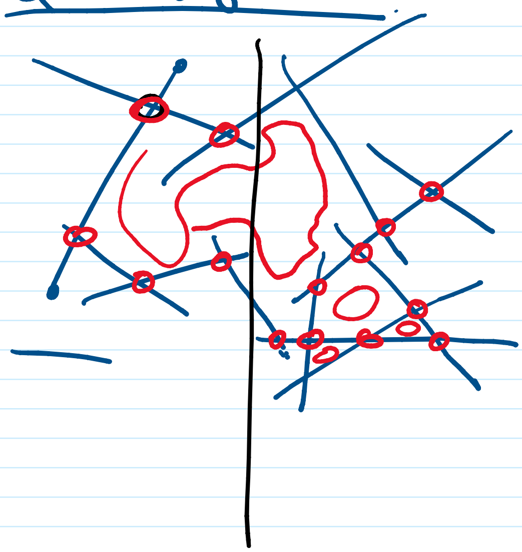
VERY SIMPLE

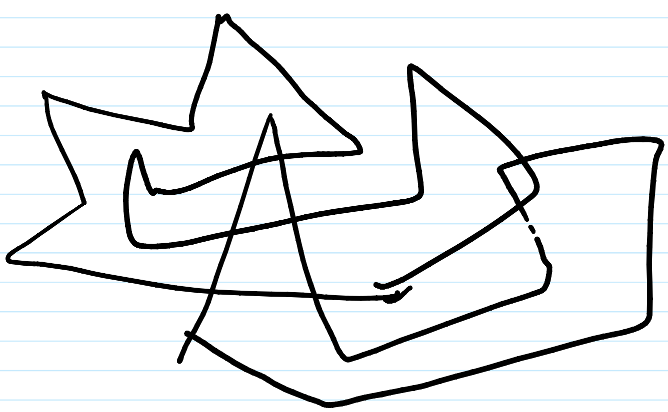
$O(d \log n)$ iterations suffice...

$$\Rightarrow \quad O(d^2 n \log n) + d^{O(d)} \log n \quad \text{expected time}$$

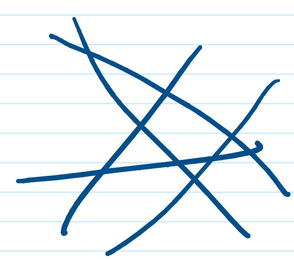
Combine $\Rightarrow$ $$O(d^2 n + \cancel{d^{O(d)}}\, d^{O(\sqrt{d})} \log n) \quad \text{expected time}$$

Line Segments: Intersections & Arrangements





Problem 1 Given n line segments, report all intersections



Problem 2 Given n line segments, construct their arrangement (subdivision into faces, edges, vertices)

Output size: $k =$ # intersections

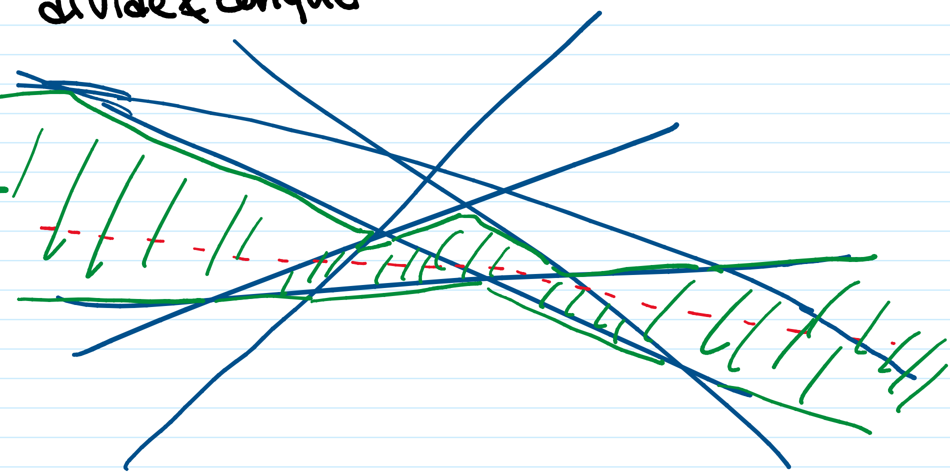
k can be $\Theta(n^2)$ in worst case

lower bound: $\Omega(n \log n + k)$

Approaches:
- incremental
- sweep
- divide & conquer

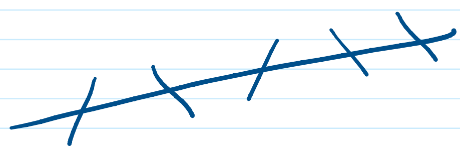
The Case of Lines:

Arrangements



$k = \Theta(n^2)$

naive alg'm:



sort intersections along each line

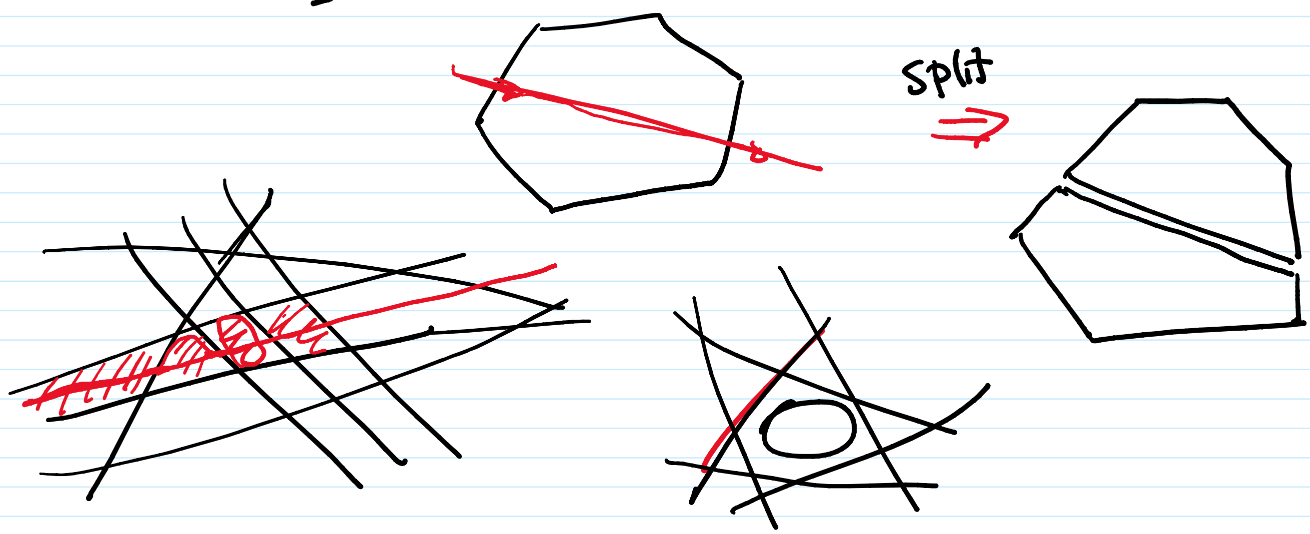
$O(n^2 \log n)$ total time

Incremental Alg'm:

0. $\mathcal{A}$ = "empty" arrangement
1. for each line ℓ {
2. locate ~~one~~ leftmost face cut by ℓ ← $O(n)$
3. generate all faces cut by ℓ by "walking" in $\mathcal{A}$
4. Split these faces in $\mathcal{A}$

3. generate all faces cut by ℓ by walking ...
4. split these faces in $\mathcal{A}$

}



Cost of lines 3-4?

naively, visit n faces
each face may be $O(n)$ vertices

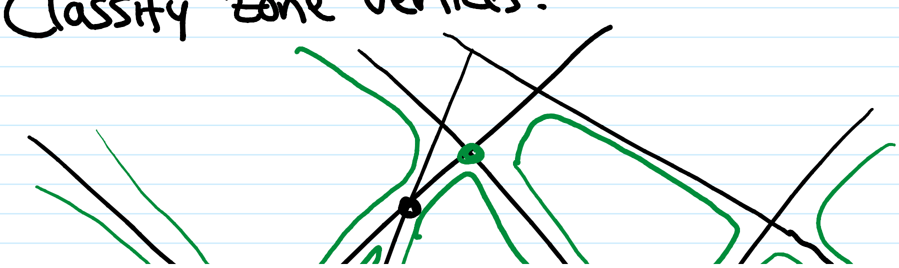
$\Rightarrow \cancel{O(n^2)}$ per iter — $O(n)$ by zone thm

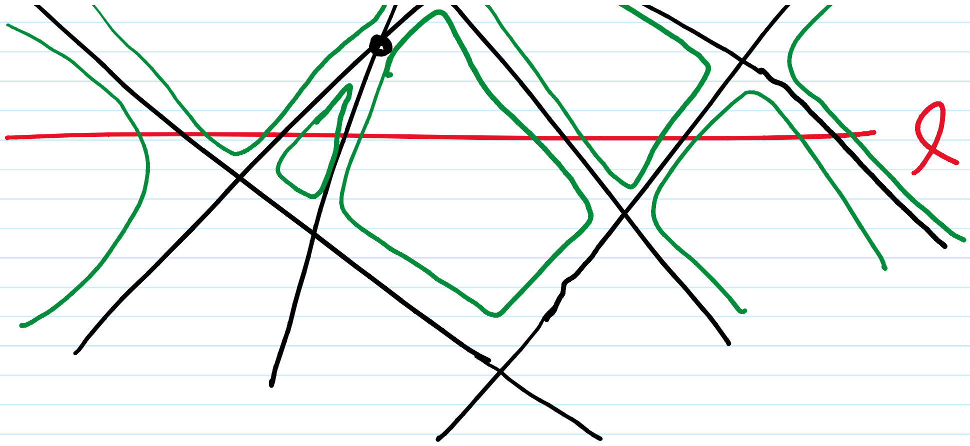
$\Rightarrow \cancel{O(n^3)}$ $O(n^2)$ total time: bad!

Thm Define the <u>zone</u> of ℓ to be collection of all faces of $\mathcal{A}$ cut by ℓ.

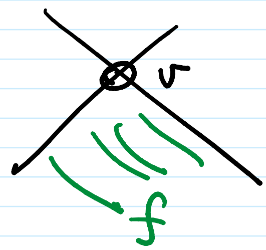
The total size of the zone is $O(n)$.

Pf: Say ℓ is horizontal.
Suffice to count zone vertices above ℓ.
Classify zone vertices:





Type 1:

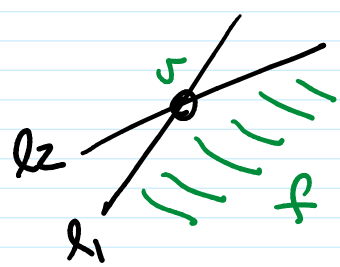


top vertex of zone face

$\Rightarrow \leq n+1$ such vertices

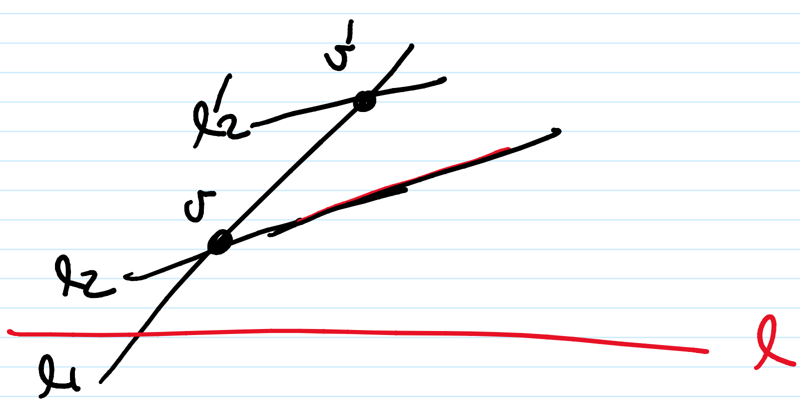
Type 2:

$\text{slope}(\ell_1) > \text{slope}(\ell_2) > 0$



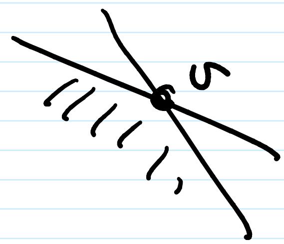
charge v to ℓ_1

Each line charged once



$\Longrightarrow \; \leq n$ such vertices

Type 3:



Symmetric

$\square$