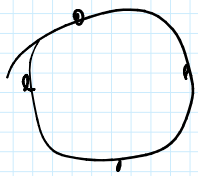
Consequences: p_l outside circle thru p_i, p_j, p_k

iff $\hat{p}_l$ above plane thru $\hat{p}_i, \hat{p}_j, \hat{p}_k$



iff $\begin{vmatrix} 1 & x_i & y_i & x_i^2+y_i^2 \\ 1 & x_j & y_j & x_j^2+y_j^2 \\ 1 & x_k & y_k & x_k^2+y_k^2 \\ 1 & x_l & y_l & x_l^2+y_l^2 \end{vmatrix} > 0$

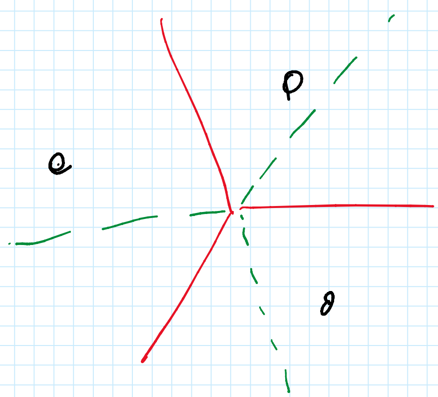
assuming p_i, p_j, p_k is ccw turn

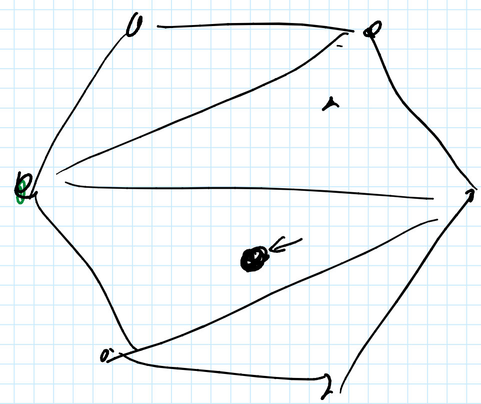
lower envelope of planes in 3D $\xleftrightarrow{\text{dual}}$ lower hull of pts in 3D

$\uparrow$ lifting transform $\qquad\qquad$ $\uparrow$ lifting

Voronoi diagram in 2D $\xleftrightarrow{\text{duality}}$ Delaunay triang in 2D

degeneracies: 4 pts co-circular





Alg'ms:

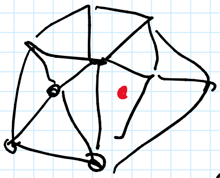
gift-wrapping $O(n^2)$

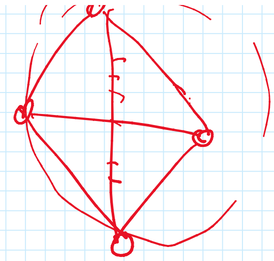
incremental $O(n^2)$ worst case

rand. $O(n \log n)$

incremental

rand. $O(n \log n)$

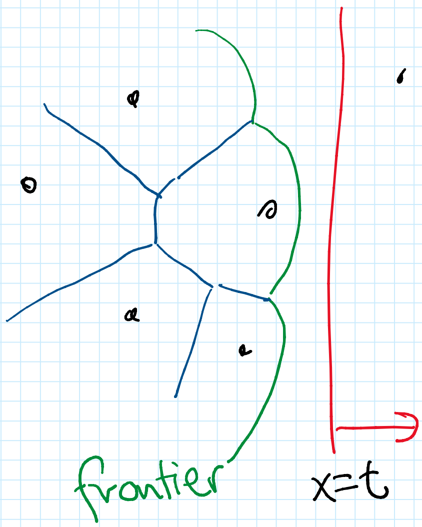


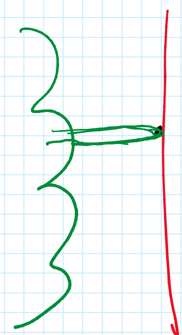


divide-and-conquer $O(n \log n)$
[Shamos, Hoey '77]

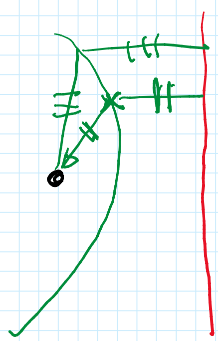
Fortune's sweepline alg'm '85: $O(n \log n)$

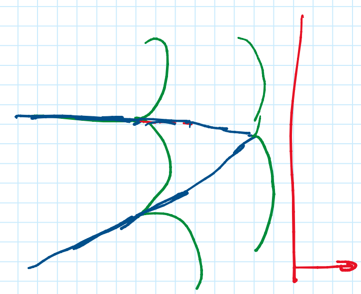
← specific to VD/DT





frontier = bisector between $P \cap \{x<t\}$ and sweep line





Applications of VD/DT

Problem 1 Closest pair of n pts in $\mathbb{R}^2$

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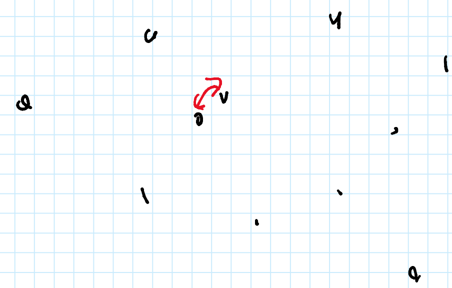
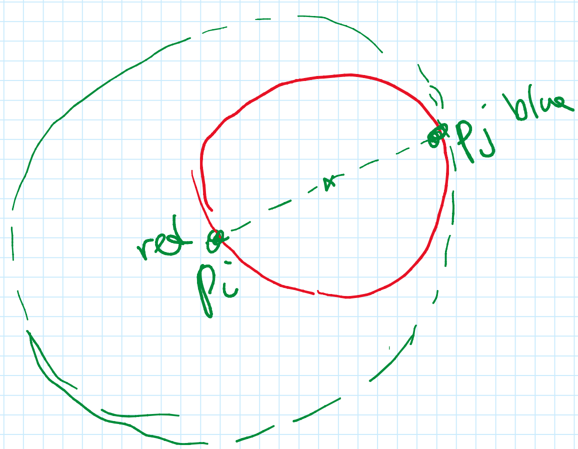


Obs If ~~$p_i\,p_j$ is closest pair~~ p_j is nearest neighbor of p_i,
then p_ip_j is a DT edge.

Pf:



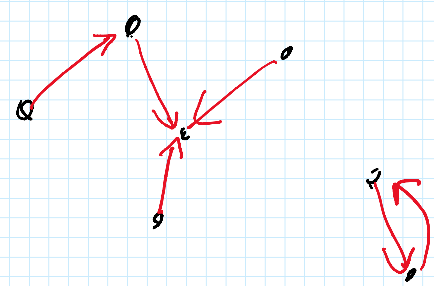
take circle thru p_i, p_j with center at midpt

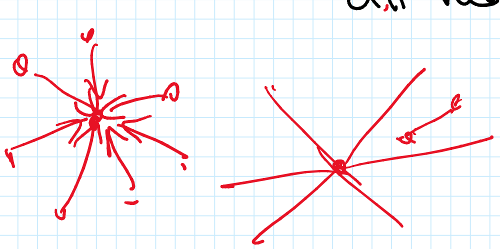
$\Rightarrow$ empty circle prop.

$\square$

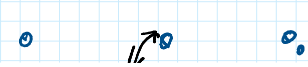
$$\Rightarrow \quad \underset{\uparrow \text{ time for DT}}{O(n\log n)} + \underset{\uparrow \text{ check all DT edges}}{O(n)} = \boxed{O(n\log n)}$$

works also for
all-nearest-neighbors

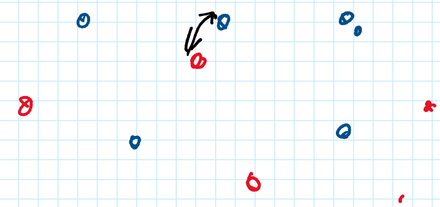




red-blue closest pair



red-blue closest pair



Problem 2

Largest empty circle
with center inside R.