3D Convex Hulls

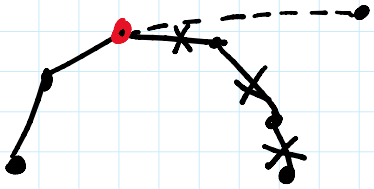
Algo0: brute force $O(n^4)$

Algo1: gift-wrapping $O(nh) \leq O(n^2)$

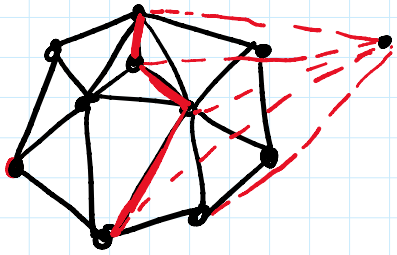
Algo 2.: Incremental. (Seidel '81)

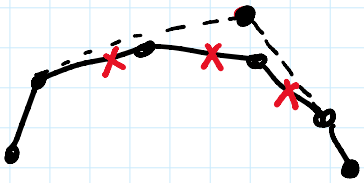
idea- insert pt one at a time

2D Graham



3D





Rough outline:

1. for $i = 4, \ldots, n$ do { // insert p_i
2. if p_i inside break
3. locate a face f visible from p_i
4. generate all faces visible from p_i by BFS/DFS
5. delete all these faces
6. Create new faces from p_i to all border edges

}

Analysis: lines 2-6 $O(n)$ time

$\Rightarrow$ $\boxed{O(n^2)}$ total time worst-case

Refinement: lines 4-5 $O($#faces destroyed$)$

Refinement: lines 4-5 $O(\#\text{faces destroyed})$
line 6 $O(\#\text{faces created})$

total # faces destroyed $\leq$ total # faces created

$\Rightarrow$ excluding lines 2-3, total work $O(\text{total \# faces created})$

Fact (Clarkson-Shor '89)

If pts are inserted in random order,

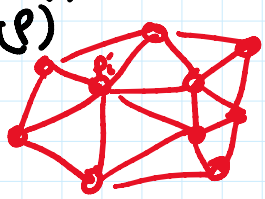
$E[\text{total \# faces created}] = O(n)$.

"backwards analysis" **Pf:** Look at n^{th} iteration.

(p_i = pt you insert in n^{th} iteration)

$\#\text{faces created} = \text{degree of } p_i \text{ in final } UH$

$UH(P - p_i)$
$UH(P)$



$E(\#\text{faces created}]$
$= \text{average degree in final } UH$
$= \frac{\text{total degree}}{n}$
$= \frac{2(\#\text{edges})}{n}$
$\leq \frac{2 \cdot 3n}{n} = 6.$

$E[\text{Total}] \leq 6n.$ □
by linearity of expectation

↑ ($T(n) \leq 6 + T(n-1)$.).

how to do lines 2-3? "point location" problem

Clarkson-Shor / Mulmuley:
$O(\log n)$ expected time
by maintaining a search structure

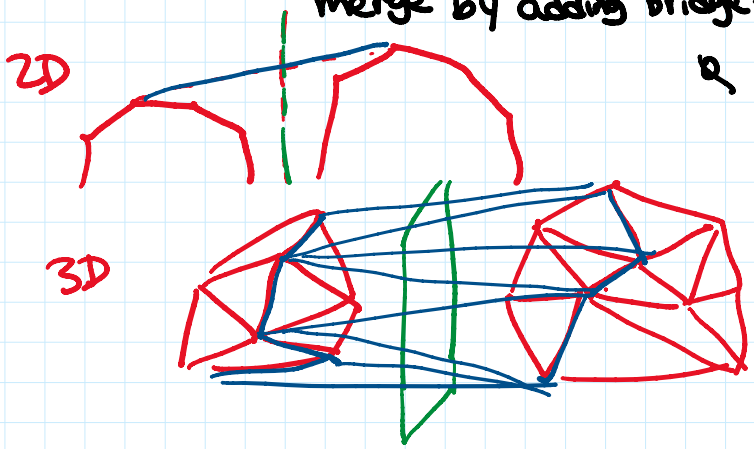
$\Rightarrow$ randomized $O(n \log n)$ expected algorithm

alg'm

(no bad input)

Algm 3: Divide & Conquer (Preparata, Hong '77)

idea- Sort in x first
compute left & right UH
merge by adding bridge-face



& doable in $O(n)$ time

$\Downarrow$

$O(n \log n)$ time

How to merge in $O(n)$ time?
"kinetic reinterpretation"