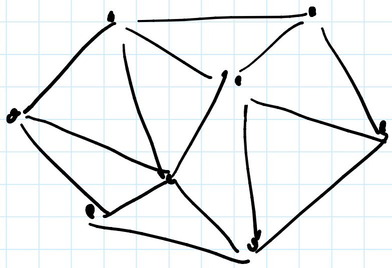
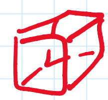
Convex Hull in 3D



n_v = # vertices
n_e = # edges
n_f = # faces

Thm $n_e, n_f = O(n_v)$.



$8-12+6=2$

Pf 1: $n_v - n_e + n_f = 2$
by Euler's formula

$4-6+4=2$

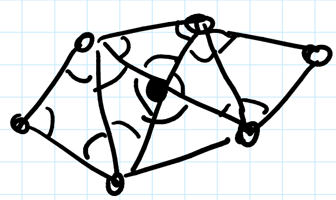
In addition, $\boxed{3n_f = 2n_e}$ (if simplicial)

$\Rightarrow \quad n_v - n_e + \frac{2}{3}n_e = 2$

$\Rightarrow \quad n_e = 3n_v - 6$

$n_f = 2n_v - 4. \quad \square$

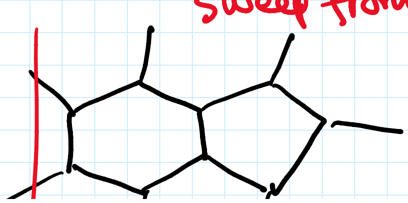
Pf 2 (Primal): look at upper hull projection
sum all angles

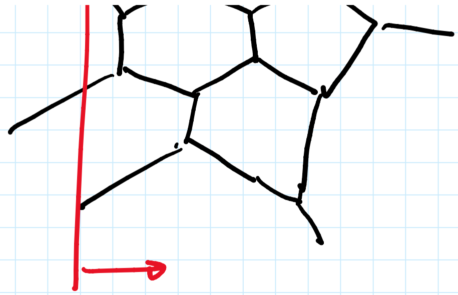


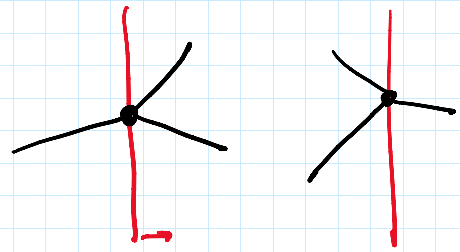
$n_f \pi = 2\pi(n_v - k) + (k-2)\pi$

$n_f = 2n_v - k - 2 < 2n_v$
$\square$

Pf 3 (Dual): look at lower envelope projection
sweep from left to right







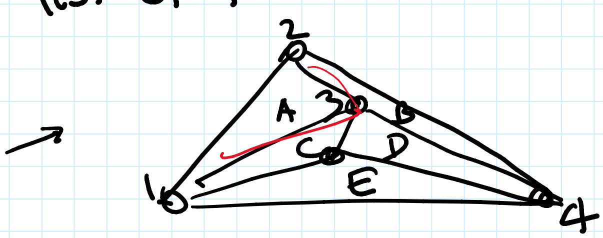
each vertex causes a face to be created/destroyed

$$n_v^* \leq 2 n_f^*$$

$$\Rightarrow \quad n_f \leq 2 n_v \qquad \square$$

Representation Options

- list of vertices (what order?)
- graph G (vertices + edges in adj lists)
- list of faces



A : 1,2,3
B : 2,3,4
C :
D :
E :

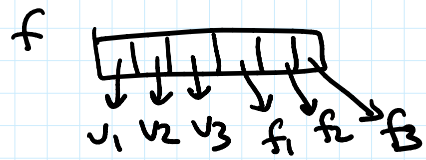
(can't navigate from face to face)

- dual graph G^*

⋮

- face-based structure (specific for triangulations)

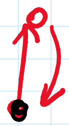
  each triangle points to 3 adj triangles

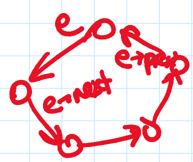


~ 12n pointers

- edge-based structures (general, for any planar subdivision)

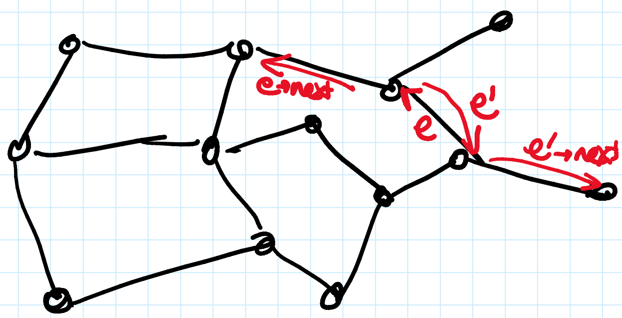
Such as doubly-connected edge list (DCEL)
quad-edged
wing-edged

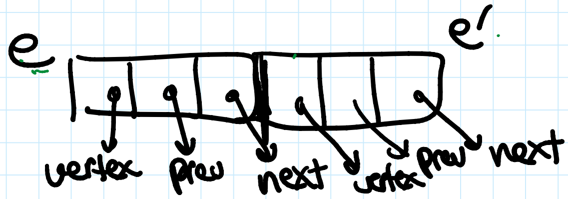




each directed edge e points

e→vertex : origin
e→next : next edge in same face (ccw)
e→prev : prev edge in same face
e' : opposite edge

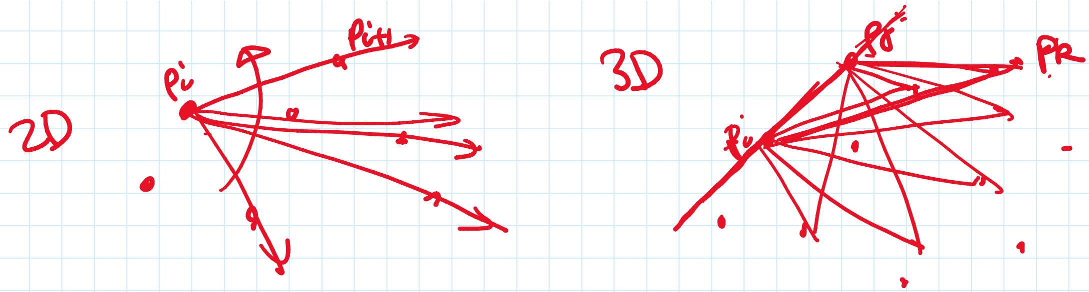


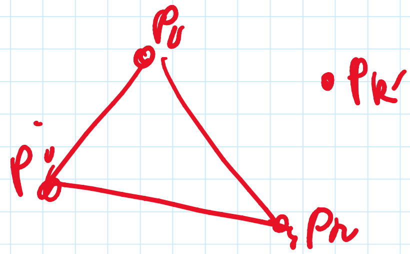


~18n pointers

Alg'm 1: Gift-Wrapping (Chand-Kapur '70)

idea- go from one CH edge to an ~~adj CH edge~~ point





- generate all edges by BFS / DFS

```
   queue/stack
Q = { an initial edge },   E = ∅
while Q ≠ ∅ {
    remove an edge pi pj from Q
    if (pi pj ∉ E) {
        add pi pj to E
        pk = any initial pt of P
        for l = 1 to n do                 ⎫
            if pl "right" of pi pj pk      ⎬ O(n)
               then pk = pl                ⎭
        add pk pj and pk pi to Q
    }
}
return E
```

Analysis: each iteration $O(n)$ time

iterations $O(h)$

hull vertices

$\Rightarrow \boxed{O(nh)}$ time