CS 498 TC, Fall 2024
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Homework 5 (due Dec 4 Wed 5pm)

Instructions: See previous homework.

1. [*30 pts*] We are given a (nonconvex) polyhedron P with n vertices in 3D (where there are no holes and all the faces are polygons). Show that we can always select a subset G of $\lfloor n/2 \rfloor$ vertices so that every point on the boundary of P is visible from some point of G. Here, we say that p and q are *visible* iff the line segment pq lies on the boundary of P.

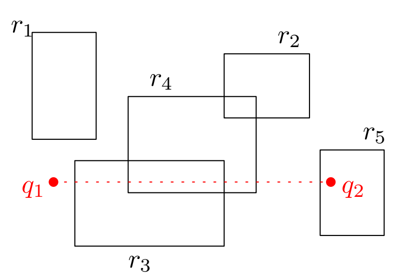
   (Hint: triangulate all the faces and apply the 4-Color Theorem for planar graphs (the boundary of P forms a planar graph)...)

2. [*50 pts*] Given a simple polygon P with n vertices and a convex polygon R with a constant number of vertices, we want to determine whether R can be placed inside P by translation, i.e., whether there exists a translated copy of R that is contained in P. Show that this problem can be solved in $O(n \log^2 n)$ time.

   (Hint: use an approach we will cover in class on the motion planning problem... Another hint: consider the complement of P.)

3. [*20 pts*] Consider the following problem: store a set S of n (axis-aligned) rectangles in 2D so that for a given query horizontal line segment $\overline{q_1q_2}$, we can quickly report all rectangles $r \in S$ that $\overline{q_1q_2}$ completely cuts across (i.e., $\overline{q_1q_2}$ intersects both the left and right side of r). Give an efficient data structure for this problem.

   (Hint: reduce the problem to orthogonal range searching. How many dimensions?)



$\overline{q_1q_2}$ completely cuts across r_3 and r_4 (but not r_5)