HW 7

Due on **Wednesday, March 22, 2023 at 9am CST**

CS 498CG: Discrete & Computational Geometry, Spring 2023 Version: **3.2**

Instructions: As in previous homeworks.

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18 (100 PTS.) Moving among rectangles.

Let R be a set of n axis aligned rectangles in the plane, not necessarily disjoint. Let s, t be two points not covered by the rectangles of R. Describe a $O(n \log^c n)$ time algorithm that decides if there a path from s to t that avoids the interior of the rectangles of R, where c is some small fixed constant. You can assume the rectangles are in general position.

19 (100 PTS.) How many containments?

Let P be a set of n points in the plane, and let R be a set of n axis aligned rectangles in the plane. A ***containment*** is when a point of $p \in P$ is contained in a rectangle $r \in R$. Assume, that for any set $R' \subseteq R$ of 10 rectangles, and any set $P' \subseteq P$ of 10 points, we have the property that there is a point $p' \in P'$, and a rectangle $r' \in R'$, such that $p' \notin r'$. Bound the maximum number of containments between points of P and R? Your bound needs to be as tight as possible (in particular, a quadratic bound is worth no points). Formally, let

$$C(P, R) = \{(p, r) \mid p \in P, r \in R, p \in r\}.$$

be the set of all containments. The number of containments is $|C(P, R)|$.

(Hint: Solve the problem for the one dimensional case of points and intervals, and then try to use segment-tree/range-tree like construction to handle the 2d case. The expected bound here is much better than quadratic.)

20 (100 PTS.) Polygons as disks.

You are given a set $\mathcal{P}$ of convex polygons in the plane, with the total number of their vertices being n. Assume that the boundary of any two such polygons intersect at most twice. Describe an algorithm, as fast as possible, that computes the boundary of the union of the polygons of $\mathcal{P}$.