HW 6

Due on **Wednesday, March 8, 2023 at 9am CST**

CS 498CG: Discrete & Computational Geometry, Spring 2023 Version: **3.0**

Instructions: As in previous homeworks.

15 (100 PTS.) Luney lune.

Do problem 9.14 in [dBCvKO08, page 217].

16 (100 PTS.) Report the points.

Do problem 10.12 in [dBCvKO08, page 241].

17 (100 PTS.) Heavy time.

Let P be a set of n points in the plane in general position. A circumcircle $C(p, q, r)$ of points $p, q, r \in P$ is ***k-heavy*** if it contains k points of P in its interior. Let $T_{\leq k}$ be the set of all circumcircles of points of P of weight at most k.

17.A. (30 PTS.)Let $C = C(p, q, r) \in T_{\leq k}$ be a given circumcircle. Let R be a random sample from P of size n/k (without repetitions). Consider the event that the triangle $\triangle = \triangle pqr$ is in the Delaunay triangulation of R (denoted by $\mathcal{D}(R)$), and let

$$\alpha(C) = \mathbb{P}[\triangle pqr \in \mathcal{D}(R)]$$

be this probability. Prove that $\alpha(C) \geq c/k^3$, for some absolute constant $c > 0$.

17.B. (30 PTS.) Prove that $\sum_{C=C(p,q,r)\in T_{\leq k}} \alpha(C) = O(n/k)$.

17.C. (40 PTS.) Combining the above two parts, prove that $|T_{\leq k}| = O(nk^2)$.

References

[dBCvKO08] Mark de Berg, Otfried Cheong, Marc J. van Kreveld, and Mark H. Overmars. *Computational Geometry: Algorithms and Applications*. Springer, Santa Clara, CA, USA, 3rd edition, 2008.