HW 5

Due on **Wednesday, March 1, 2023 at 9am CST**

CS 498CG: Discrete & Computational Geometry, Spring 2023 Version: **2.0**

Instructions: As in previous homeworks.

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12 (100 PTS.) Stab the segments.

12.A. (50 PTS.) Given a set S of n segments in the plane, describe an $O(n)$ (expected) time algorithm that decides if there is a single line that stabs all the segments

If to get this running time you need to assume something on the segments, what is your assumption? Try to use an assumption that is as general as possible.

12.B. (50 PTS.) Assuming that there is no single line that stabs all the segments of S, describe an algorithm, as fast as possible, that outputs a line that stabs the largest number of segments of S. What is the running time of your algorithm? Explain/prove the correctness of your algorithm.

13 (100 PTS.) A not so kosher question.

Let L be a set of $n = 2k + 1$ lines in general position in the plane. A point p of $\mathcal{A}(L)$ is ***halving*** if it has exactly k lines vertically below it, and k lines vertically above it (i.e., a halving point lies on one of the liens of L). The *closure* of all the halving points is the ***middle level*** of L. The middle level is a polygonal line that starts and ends with two rays that lie on the same line (why?).

13.A. (50 PTS.) Let L' be another set of $n' = 2k' + 1$ lines, such that $L \cup L'$ are in general position. Prove that the two halving levels of L and L', respectively, must intersect. Describe an efficient algorithm for computing a point that lies in this intersection (there might be several such points). What is the running time of your algorithm as a function of $m = |L| + |L'|$?

13.B. (50 PTS.) Let P and Q be two sets of points of odd size in the plane (assume that $P \cup Q$ is in general position). Prove (using the previous part), that there is a line ℓ that passes through a point $p \in P$ and a point $q \in Q$, such that ℓ has $(|P| - 1)/2$ points of P on its two sides, and $(|Q| - 1)/2$ points of Q on its two sides.

14 (100 PTS.) Reflect through the edges.

Do problem 7.19 in the book (page 171).