HW 4

Due on **Wednesday, February 22, 2023 at 9am CST**

CS 498CG: Discrete & Computational Geometry, Spring 2023 Version: **1.0**

Instructions: As in previous homeworks.

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9 (100 PTS.) Fractional cascading.

Reading section 5.6 in the book [dBCvKO08], and do question 5.10 (page 119).

10 (100 PTS.) Intersecting segments.

Let S be a set of n segments in the plane, and assume that there are k pairs of them that intersect.

10.A. (30 PTS.) Let $\Pi = \pi_1, \ldots, \pi_n$ be a uniform random permutation of S, and let $\Pi_i = \{\pi_1, \ldots, \pi_i\}$ be the first i segments. Provide an exact formula on the expected number of vertices and edges of the arrangement $\mathcal{A}(\Pi_i)$. (Hint: What is the exact probability that a vertex $v = s \cap s'$ appears in this arrangement?)

In particular, what is the expected complexity of the vertical decomposition of $\mathcal{A}(\Pi_i)$.

10.B. (70 PTS.) Using the first part, analyze the randomized incremental algorithm that computes the vertical decomposition of $\mathcal{A}(S)$ in $O(n \log n + k)$ expected time. Prove your bounds.

11 (100 PTS.) Point location on triangles.

Let T be a set of n triangles in three dimensions. Describe how to build a data structure (in polynomial time), such that given a query point q, one can decide in $O(\log n)$ time, which is the first triangle that lies above q if we shoot a vertical ray shooting.

(Observe, that we do not care about the preprocessing time and space used as long as they are polynomial. In particular, try to come up with simple schemes to solve this problem. Googling would probably yields solutions that are unnecessarily complicated.)

References

[dBCvKO08] Mark de Berg, Otfried Cheong, Marc J. van Kreveld, and Mark H. Overmars. *Computational Geometry: Algorithms and Applications*. Springer, Santa Clara, CA, USA, 3rd edition, 2008.