HW 3

Due on **Wednesday, February 15, 2023 at 9am CST**

CS 498CG: Discrete & Computational Geometry, Spring 2023 Version: **1.0**

Instructions: As in previous homeworks.

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7 (100 PTS.) kth distance.

Let P be a set of n points in the plane, and consider the set of distances

$$D(P) = \{\|pq\| \mid p, q \in P, p \neq q\}.$$

Assume that all the pairwise distances in P are distinct.

7.A. (80 PTS.) Let $k > 0$ be a given integer number (think about k as being small compared to n). Present an algorithm that in $O(n)$ expected time, computes a set $X \subseteq P$ of $O(k)$ points, such that the kth smallest value in $D(P)$ and $D(X)$ is the same.

(Hint: Extend the alternative algorithm seen in class for the closest pair.)

7.B. (20 PTS.) Given P and k, show how to compute the kth smallest value in $D(P)$ in $O(n+k^2)$ time (faster algorithms are known, but they are significantly more complicated).

8 (100 PTS.) Approximate cover.

Let C and P be two given sets of points in the plane, such that $k = |C|$ and $n = |\mathsf{P}|$. Let $r = \max_{p \in \mathsf{P}} \min_{c \in C} \|cp\|$ be the ***covering radius*** of P by C (i.e., if we place a disk of radius r around each point of C, all those disks cover the points of P).

Give an $O(n + k \log n)$ expected time algorithm that outputs a number α, such that $r \leq \alpha \leq 2r$.

(Hint: Extend the alternative algorithm seen in class for the closest pair.)

Some other problems

(Not for submission.)

- (100 PTS.) Counting intersections

  Do question 5.11 from [dBCvKO08] – page 119.

References

[dBCvKO08] Mark de Berg, Otfried Cheong, Marc J. van Kreveld, and Mark H. Overmars. *Computational Geometry: Algorithms and Applications*. Springer, Santa Clara, CA, USA, 3rd edition, 2008.