HW 2

Due on **Wednesday, February 8, 2023 at 9am CST**

CS 498CG: Discrete & Computational Geometry, Spring 2023 Version: **5**

Instructions: As in previous homeworks.

4 (100 PTS.) Shallow triangulation.

Do question 3.14 from [dBCvKO08] – page 61.

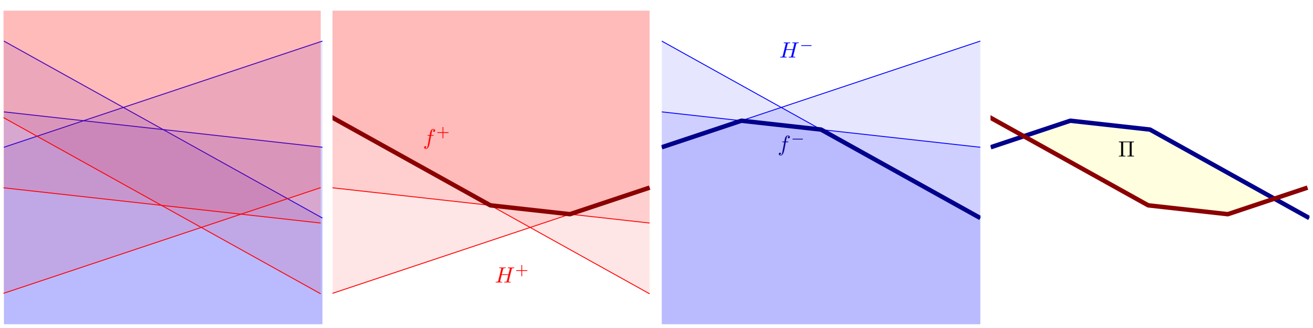
5 (100 PTS.) Point location in a triangulation of a star polygon.

5.A. (50 PTS.) A ***star polygon*** is a polygon P that has a point $p \in \Pi$, such that for any point $q \in P$, we have that $pq \subseteq P$. Describe a linear time algorithm for deciding if a given polygon is a star polygon.

5.B. (50 PTS.) Show how to preprocess a star polygon Π into a data-structure, such that given a query point q, one can decide in $O(\log n)$ time if $q \in \Pi$.

6 (100 PTS.) Linear programming in 2d

Given a set H of n halfplanes in two dimensions (in general position), extend the algorithm seen in class (for finding a vertex about a set of lines), to compute a point that belong to all these halfplanes, if such a point exists. In particular, you have to describe a deterministic linear time algorithm to compute such a point. To this end, answer the following:



Formally, let $\Pi = \cap_{h \in H} h$ be the polygon forming the feasible region.

6.A. (30 PTS.) Let H^+ and H^- be the partition of H into halfplanes containing the positive and negative infinite y-axis (i.e., halfplanes above the lines that bounds them, and the ones below), respectively. Let L^+ and L^- be the set of lines bounding the halfplanes of H^+ and H^-, respectively. Let

$$f^+(x) = \max_{\ell \in L^+} \ell(x) \qquad \text{and} \qquad f^-(x) = \min_{\ell \in L^-} \ell(x).$$

Prove that $g(x) = f^-(x) - f^+(x)$ is a concave function, and one can compute $g(\alpha)$ and $g'(\alpha)$ in linear time, for a specified value α. Furthermore, the vertical line $x = \beta$ intersects $\Pi \iff g(x) \geq 0$.

6.B. (40 PTS.) (Prune.) Present a linear time algorithm that either computes (i) A feasible point $p \in \Pi$, or (ii) a subset $H' \subseteq H$, such that the lowest point (in the y value) in $\cap_{h \in H'} h$ and Π is the same, and $|H'| \leq cn$, where $c < 1$.

6.C. (30 PTS.) Using previous parts, present a deterministic linear time algorithm for computing a point $p \in \Pi$ with minimum y-value or output that Π is empty.

References

[dBCvKO08] Mark de Berg, Otfried Cheong, Marc J. van Kreveld, and Mark H. Overmars. *Computational Geometry: Algorithms and Applications*. Springer, Santa Clara, CA, USA, 3rd edition, 2008.