LECTURE 1, Jan 21st, 2025

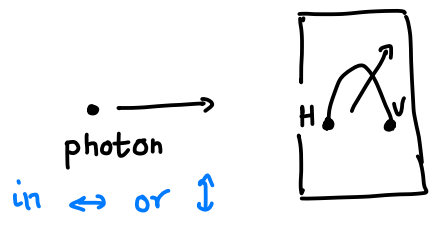
What is a qubit?

A bit : $\{0,1\}$ ← two discrete values

How can we physically represent a bit?

Low voltage	Spin of an electron : up	Polarization of a photon : ↔ horizontal $\|0\rangle$
High voltage	down	↕ vertical $\|1\rangle$

Only way to know the state via measuring device



tells you whether photon is ↔ or ↕

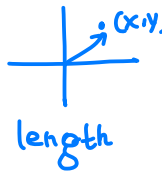
QM Law 1 If a "particle" can be in one of 2 basic states $|0\rangle$ or $|1\rangle$ then it can also be in a superposition state, meaning

"α amplitude on $|0\rangle$, β amplitude on $|1\rangle$"
where α, β are complex numbers satisfying $|\alpha|^2 + |\beta|^2 = 1$

Recall

$z = x + iy$ complex number where $i = \sqrt{-1}$

$|z| = \sqrt{x^2+y^2}$



Simplest quantum system with two degrees of freedom

Such a state is called a **qubit**.

We can represent it by a vector $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ ← unit vector since $|\alpha|^2 + |\beta|^2 = 1$

E.g. a photon may have the state "$\frac{1}{\sqrt{2}}$ amplitude on $|0\rangle$, $\frac{1}{\sqrt{2}}$ amplitude on $|1\rangle$" $\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$

OR $\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} + \frac{1}{2} = 1$

"$\frac{\sqrt{-1}}{\sqrt{2}}$ amplitude on $|0\rangle$, $-\frac{1}{\sqrt{2}}$ amplitude on $|1\rangle$" $\begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$

OR $\left|\frac{i}{\sqrt{2}}\right|^2 + \left(\frac{-1}{\sqrt{2}}\right)^2 = \frac{1}{2} + \frac{1}{2} = 1$

"1 amplitude on $|0\rangle$, 0 amplitude on $|1\rangle$" $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

called "$|0\rangle$"

NEXT LECTURE

You cannot read a quantum state, i.e., access α, β directly
Only way to extract information is via measurement