What is a qubit?

A bit: \{0, 1\} \rightarrow two discrete values

How can we physically represent a bit?

<table>
<thead>
<tr>
<th>Low voltage</th>
<th>Spin of an electron: up</th>
<th>Polarization of a photon: \leftrightarrow horizontal</th>
</tr>
</thead>
<tbody>
<tr>
<td>High voltage</td>
<td>Spin of an electron: down</td>
<td>Polarization of a photon: \downarrow vertical</td>
</tr>
</tbody>
</table>

Only way to know the state via measuring device tells you whether photon is \leftrightarrow \$ or \$ state

QM law 1 If a "particle" can be in one of 2 basic states \(10\) or \(1\) then it can also be in a superposition state, meaning:

" \alpha \) amplitude on \(10\), \( \beta \) amplitude on \(1\)"
where \( \alpha, \beta \) are complex numbers satisfying \( |\alpha|^2 + |\beta|^2 = 1 \)

Simplest quantum system with two degrees of freedom

Such a state is called a **qubit**.

We can represent it by a vector \( \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \leftrightarrow \) unit vector since \( |\alpha|^2 + |\beta|^2 = 1 \)

E.g. a photon may have the state " \( \frac{1}{\sqrt{2}} \) amplitude on \(10\), \( \frac{1}{\sqrt{2}} \) amplitude on \(1\)"

\[ \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \]

\[ \text{OR} \]

\[ \left( \frac{1}{\sqrt{2}} \right)^2 + \left( \frac{1}{\sqrt{2}} \right)^2 = \frac{1}{2} + \frac{1}{2} = 1 \]

" \( \frac{1}{\sqrt{2}} \) amplitude on \(10\), \( -\frac{1}{\sqrt{2}} \) amplitude on \(1\)"

\[ \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \]

\[ \text{OR} \]

\[ \left( \frac{1}{\sqrt{2}} \right)^2 + \left( -\frac{1}{\sqrt{2}} \right)^2 = \frac{1}{2} + \frac{1}{2} = 1 \]

"1 amplitude on \(10\), 0 amplitude on \(1\)"

\[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \]

called "\(10\)"

You cannot read a quantum state, i.e., access \( \alpha, \beta \) directly

Only way to extract information is via measurement