Program Verification: Lecture 18

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University of Illinois at Urbana-Champaign

The Mathematical Model of a Rewrite Theory

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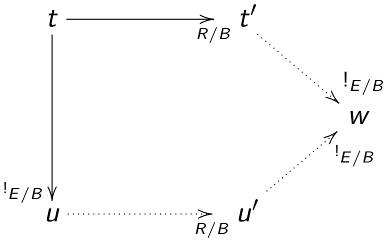

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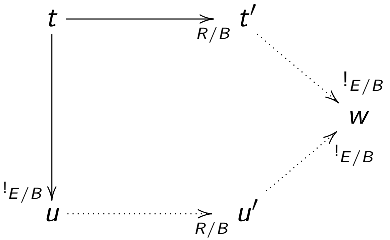

Maude's Coherence Checker tool checks this property.

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Maude tools can be used to verify that $\mathbb{C}_{\mathcal{R}} \models \varphi$ in logics (1)–(3).

In this course we shall verify properties of $\mathbb{C}_{\mathcal{R}}$ in both modal logic and (linear time) temporal logic (LTL) by:

- Explicit-state model checking.
- Symbolic model checking.
- Symbolic model checking + Theorem proving.

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Following Saul Kripke, analytic philosophers model this with a so-called possible worlds semantics.

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That is, the transition system on which we give a Kripke semantics to φ is $(\mathbb{C}_{\Sigma/\vec{E},B,St}, \to_{\mathbb{C}_{\mathcal{R}}})$,

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Property $u|\varphi$ is available to Maude users: in Maude's `search` command $u|\varphi$ is specified as the target pattern with syntax:

u `such that` φ `.`

[1] φ could be allowed to be a $\Sigma \cup \Sigma_{aux}$-formula with Σ_{aux} auxiliary functions.

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We can apply all this to verify invariants $\mathcal{R}, init \models \Box p^Q$ (resp. reachable sets, $\mathcal{R}, init \models \Diamond p^Q$) by verifying through explicit-state model checking that $\mathcal{R}, init \not\models \Diamond\neg p^Q$ (resp. $\mathcal{R}, init \models \Diamond\neg p^Q$) using Maude's `search` command.

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Let us illustrate this explicit-state model checking method with `QLOCK`, a mutual exclusion protocol proposed by K. Futatsugi, where the number of processes is unbounded.

The QLOCK Mutual Exclusion Protocol

```
mod QLOCK is protecting NAT .
 sorts NatMSet NatList State .
 subsorts Nat < NatMSet NatList .
 op mt : -> NatMSet [ctor] .
 op _ _ : NatMSet NatMSet -> NatMSet [ctor assoc comm id: mt] .
 op nil : -> NatList [ctor] .
 op _;_ : NatList NatList -> NatList [ctor assoc id: nil] .
 op {_<_|_|_|_>} : NatMSet NatMSet NatMSet NatMSet NatList -> State [ctor] .
  op [_] : Nat -> NatMSet .  *** set of first n numbers
 op init : Nat -> State .   *** initial state, parametric on n

 vars n i j : Nat .  vars S U W C : NatMSet .  var Q : NatList .
 eq [0] = mt .
 eq [s(n)] = n [n] .
 eq init(n) = {[n] < mt | mt | mt | nil >} .

 rl [join] : {S i < U | W | C | Q >} => {S < U i | W | C | Q >} .
 rl [n2w] : {S < U i | W | C | Q >} => {S < U | W i | C | Q ; i >} .
 rl [w2c] : {S < U | W i | C | i ; Q >} => {S < U | W | C i | i ; Q >} .
 rl [c2n] : {S < U | W | C i | i ; Q >} => {S < U i | W | C | Q >} .
 rl [exit] : {S < U i | W | C | Q >} => {S i < U | W | C | Q >} .
endm
```

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from, e.g., the initial state `init(7)` with seven processes.

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Consider the following specification of a readers-writers system.

Bounded Model Checking of Invariants (III)

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mod R&W is
  protecting NAT .
  sort Config .
  op <_,_> : Nat Nat -> Config [ctor] .   --- readers/writers

  vars R W : Nat .

  rl < 0, 0 > => < 0, s(0) > .
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- mutual exclusion: readers and writers never access the resource simultaneously: only readers or only writers can do so at any given time.
- one writer: at most one writer will be able to access the resource at any given time.
- deadlock freedom: there are no deadlocks.

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- mutual exclusion: readers and writers never access the resource simultaneously: only readers or only writers can do so at any given time.
- one writer: at most one writer will be able to access the resource at any given time.
- deadlock freedom: there are no deadlocks.

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However, since the number of readers can grow unboundedly, Maude's `search` commands to find counterexamples instantiating either of these two patterns from `< 0, 0 >` search forever.

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No solution.
states: 1000002  rewrites: 2000001 in 36480ms cpu (50317ms real)

Maude> search [1, 1000000] < 0,0 > =>* < N:Nat, s(s(M:Nat)) > .
No solution.
states: 1000002  rewrites: 2000001 in 38910ms cpu (41650ms real)

Maude> search [1, 1000000] < 0,0 > =>! C:Config .
No solution.
states: 1000003  rewrites: 2000002 in 5752ms cpu (5821ms real)
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Thus verifying these three invariants up to depth 10^6.