CS 476 Homework #4 Due 10:45am on 10/22

Note: Answers to the exercises listed below (in typewritten form, preferably using Latex) should be emailed by the above deadline to `meseguer@illinois.edu`.

1. Prove **Ex.11.6** In Lecture 11.
2. Prove **Ex.13.1** in Lecture 13.
3. Consider the equational theory (Σ, E) defined by the functional module:

```
fmod PEANO-p is
sorts NzNat Nat .
subsort NzNat < Nat .
op 0 : -> Nat [ctor] .
op s : Nat -> NzNat [ctor] .
op p : NzNat -> Nat .
eq p(s(N:Nat)) = N:Nat .
endfm
```

   which defines the predecessor function `p`. The term rewriting system $(\Sigma, \vec{E})$ is sort-decreasing, confluent, terminating, and sufficiently complete w.r.t. $\Omega = \{0, s\}$. This can be easily checked by hand or using the Maude Formal Environment and the MTA tool; *you can assume that all these properties hold in what follows.* By the theorem in pg. 16 of Lecture 13, this means that for `PEANO-p` we have a Σ-isomorphism $\mathbb{T}_{\Sigma/E} \cong \mathbb{C}_{\Sigma/\vec{E}}$ between the initial (Σ, E)-algebra $\mathbb{T}_{\Sigma/E}$ and the canonical term algebra $\mathbb{C}_{\Sigma/\vec{E}}$.

   The point of this exercise is for you to verify for `PEANO-p` that we can have a Σ-equation $u = v$ such that $u = v$ is an inductive theorem of (Σ, E), i.e., $(\Sigma, E) \models_{ind} u = v$, but $E \not\vdash u = v$.

   Using the fact that $\mathbb{T}_{\Sigma/E} \cong \mathbb{C}_{\Sigma/\vec{E}}$, do the following:

   (a) Prove that $E \not\vdash s(p(y\!:\!NzNat)) = y\!:\!NzNat$.

   (b) Prove that $(\Sigma, E) \models_{ind} s(p(y : NzNat)) = y : NzNat$. **Hint**: You may consider using the theorem characterizing the inductive theorems of a theory (Σ, E) stated in pg. 6 of Lecture 14.