Program Verification: Lecture 25

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Simulation and Bisimulation Maps of Transition Systems

Given two transition systems $\mathcal{A} = (A, \rightarrow_{\mathcal{A}})$ and $\mathcal{B} = (B, \rightarrow_{\mathcal{B}})$, a simulation map f from $\mathcal{A}$ to $\mathcal{B}$, denoted $f : \mathcal{A} \rightarrow \mathcal{B}$, is a function $f : A \rightarrow B$ that is "transition preserving" in the sense that any transition $a \rightarrow_{\mathcal{A}} a'$ in $\mathcal{A}$ is mapped by f to a corresponding transition $f(a) \rightarrow_{\mathcal{B}} f(a')$ in $\mathcal{B}$.

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A simulation map $f : \mathcal{A} \to \mathcal{B}$ is called a bisimulation iff, in addition, for any state of the form $f(a) \in B$ and any transition $f(a) \to_{\mathcal{B}} b$ there exists and $a' \in A$ and transition $a \to_{\mathcal{A}} a'$ such that $f(a') = b$.

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Given a transition system $\mathcal{A} = (A, \to_{\mathcal{A}})$ and subsets $U, V \subseteq A$, we are interested in the reachability property:

$$\exists x \in U,\ \exists y \in V,\ x \to_{\mathcal{A}}^{*} y$$

which we abbreviate to $\exists U \to^{*} V$. If this property holds for specific $U, V \subseteq A$ we write: $\mathcal{A} \models \exists U \to^{*} V$.

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which we abbreviate to $\exists U \to^* V$. If this property holds for specific $U, V \subseteq A$ we write: $\mathcal{A} \models \exists U \to^* V$. Note that $\mathcal{A} \not\models \exists U \to^* V$ iff $\forall x \in U,\ \forall y \in V,\ x \not\to^*_{\mathcal{A}} y$ holds in $\mathcal{A}$, abbreviated $\mathcal{A} \models \forall U \not\to^* V$.

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The proofs of these two theorems are given in the Appendix.

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Let $f : \mathcal{A} \rightarrow \mathcal{B}$ be a simulation map, then for any $U, V \subseteq A$, $\mathcal{A} \models \exists U \rightarrow^{*} V$ implies $\mathcal{B} \models \exists f(U) \rightarrow^{*} f(V)$. Equivalently, $\mathcal{B} \models \forall f(U) \not\rightarrow^{*} f(V)$ implies $\mathcal{A} \models \forall U \not\rightarrow^{*} V$.

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Note the for $U, I \subseteq A$, I is an invariant from U iff $\mathcal{A} \models \forall U \not\to^* A \setminus I$. Thus, we can verify the invariant by proving $\mathcal{B} \models \forall f(U) \not\to^* f(A \setminus I)$.

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Equational Abstractions

Simulation and bisimulation maps can be very useful to verify properties of concurrent systems specified as (not necessarily topmost) rewrite theories $\mathcal{R} = (\Sigma, E \cup B, R)$, not by reasoning directly on $\mathcal{R}$, but by shifting our ground and reasoning on a quotient of $\mathcal{R}$.

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The following theorem follows trivially from the fact that if $t \rightarrow_{R/E \cup B} t'$, then, a fortiori, $t \rightarrow_{R/E \cup B \cup G} t'$.

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Theorem

Given a rewrite theory $\mathcal{R} = (\Sigma, E \cup B, R)$, Σ-equations $G = E' \cup B'$, and a top sort State, the unique surjective Σ-homomorphism $[\_]_{E \cup B \cup G} : \mathbb{T}_{\Sigma/E \cup B} \rightarrow \mathbb{T}_{\Sigma/E \cup B \cup G}$ induces a simulation map $[\_]_{E \cup B \cup G} : (T_{\Sigma/E \cup B, State}, \rightarrow_{R/E \cup B}) \rightarrow (T_{\Sigma/E \cup B \cup G, State}, \rightarrow_{R/E \cup B \cup G})$.

Equational Abstractions (II)

Equational abstractions can make the set of reachable states from an initial state *init* finite. In this way, invariants and, more generally, LTL properties that cannot be verified by explicit-state model checking can be verified using an equational abstraction $\mathcal{R}/G$.

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In what follows we shall focus on the use of equational abstractions for symbolic model checking. Therefore, I will assume a topmost rewrite theory $\mathcal{R} = (\Sigma, E \cup B, R)$ such that $E \cup B$ is FVP. We shall then be interested in equational abstractions of the form $\mathcal{R}/G$, where $G = E' \cup B'$ is such that $E \cup E' \cup B \cap B'$ is also FVP modulo $B \cap B'$.

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Since for each pattern term with variables p, the quotient homomorphism $[\_]_{E \cup B \cup G} : \mathbb{T}_{\Sigma/E \cup B}(X) \to \mathbb{T}_{\Sigma/E \cup B \cup G}(X)$ maps each $[p]_{E \cup B}$ to $[p]_{E \cup B \cup G}$, p in $\mathcal{R}/G$ just describes the image under $[\_]_{E \cup B \cup G}$ of p in $\mathcal{R}$ as the symbolic description of the set $[\![p]\!]_{\mathcal{R}}$ of all $E \cup B$-equivalence classes of ground instances of p, which is just $[\![p]\!]_{\mathcal{R}/G}$.

Equational Abstractions (III)

In particular, if the complement of an invariant I in $\mathcal{R}$ is symbolically described by a finite set of pattern terms $p_1, \ldots, p_k$, in case the symbolic state space to reach an instance of some p_i from a symbolic initial state u is infinite, we can use a topmost equational abstraction $\mathcal{R}/G$ whose equations are FVP to try to make the symbolic search space finite.

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Then, by the first Theorem in pg. 3 of this 3 of this lecture, we can use symbolic model checking from a symbolic initial state u to show in $\mathcal{R}/G$ that $\forall u \not\to^* p_i$, $1 \leq i \leq k$. However, in some cases we might get some spurious counterexample.

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But by the second Theorem in page 3 of this lecture, no spurious counterexamples will exist if the homomorphism $[\_]_{E \cup B \cup G} : \mathbb{T}_{\Sigma/E \cup B} \to \mathbb{T}_{\Sigma/E \cup B \cup G}$ actually defines a bisimulation. I shall focus on bisimulations in what follows.

Bisimilar Equational Abstractions

We say that an equational abstraction $\mathcal{R}/G$ defines an bisimilar equational abstraction of $\mathcal{R}$ iff the simulation map

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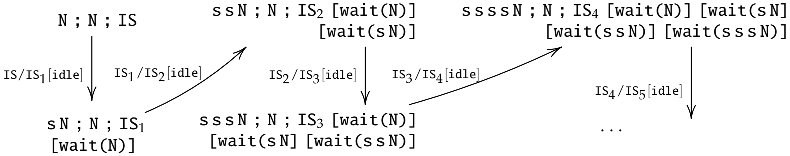
Theorem

Let $\mathcal{R} = (\Sigma, E \cup B, R)$ be a topmost rewrite theory such that $G = E \cup B$ is FVP, and $G' = E' \cup B'$ is such that $E \cup E' \cup B \cup B'$ is FVP modulo $B \cup B'$. $\mathcal{R}/G'$ defines a bisimilar equational abstraction of $\mathcal{R}$ if for each $(u_0^i = u_1^i) \in G'$, $1 \leq i \leq p$, and $(t_0^j \rightarrow t_1^j) \in R$, $1 \leq j \leq q$, and each $\sigma \in Unif_G(t_{b'}^j = u_b^i)$, $0 \leq b \leq 1$, $0 \leq b' \leq 1$, there exists a θ such that $u_{b' \oplus 1}^i \sigma =_G t_b^j \theta \ \wedge \ t_{b \oplus 1}^j \theta =_G t_{b \oplus 1}^j \sigma$, where $\oplus$ denotes exclusive or.

Bakery Algorithm: Infinite-State for some Initial States



(Infinite Folding Logical Transition System : infinite initial state - infinite state space)

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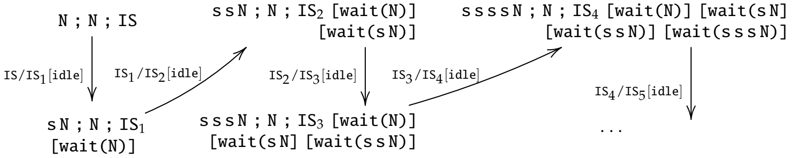

(Infinite Folding Logical Transition System : infinite initial state - infinite state space)

- Many verification problems for infinite-state systems are due to unbounded number of processes
- All approaches use a symbolic finite representation of an infinite number of processes
- Bisimulation proofs written by hand or hard to reuse

An Equational Abstraction of the Bakery Algorithm

- For our bakery protocol we can obtain a bisimilar equational abstraction by restricting the abstraction only to the following equation G', which intuitively collapses extra waiting processes that does not introduce any new behaviors:

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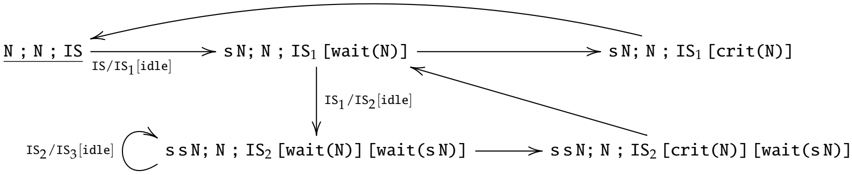

(Abstract Bisimilar Folding Logical Transition System)