CS 476 Homework #8 Due 10:45am on 10/20

Note: Answers to the exercises listed below should be given in *typewritten form* (latex formatting preferred) by the deadline mentioned above. You should email your answers and also all the Maude code and all screenshots of your tool interactions for solving Problem 2 to `nishant2@illinois.edu`.

1. Solve Exercise 100 in *STACS*.

   Note. Your equality proofs should *only use the axioms of the theory of groups listed in Exercise 100* in *STACS* and no other equations. However, equations already proved using such axioms can also be used as *lemmas* to prove other equations.

2. Consider the following module [available in the Latex version of this homework], specifying an alternative axiomatization of the *theory of groups*, where `i` denotes the *inverse* operation $(\_)^{-1}$

```
set include BOOL off .

fth GROUP is
  sort Group .
  op 1 : -> Group .
  op i : Group -> Group .
  op _*_ : Group Group -> Group .

  var x y z : Group .
  eq  (x * y) * z    = x * (y * z) .
  eq  1 * x          = x .
  eq  x * 1          = x .
  eq  x * i(x)       = 1 .
  eq  i(x) * x       = 1 .
  eq  i(1)           = 1 .
  eq  i(i(x))        = x .
  eq  i(x * y)       = i(y) * i(x) .
  eq  x * (i(x) * y) = y .
  eq  i(x) * (x * y) = y .
endfth
```

   Do the following:

   (a) Prove that the module `GROUP` is locally confluent by using the Church-Rosser Checker tool.

   (b) Use the MTA tool to prove the termination of `GROUP`.
   Hint. This module has a termination poof by an $A \vee C$-RPO order.

   (c) Call Σ_G the signature of the theory of groups, E_G the axiomatization of the theory of groups in Exercise 100 in *STACS*, and E'_G the equations in the above theory `GROUP`. Prove the theory equivalence:

   $$(\Sigma_G, E_G) \equiv (\Sigma_G, E'_G)$$

   Hint. To prove (c) more easily you may find it useful to use Maude to automate part of the proof, taking advantage of the fact that, as you have already proved in parts (a) and (b), $\vec{E}'_G$ is confluent and terminating.