CS 476 Homework #8 Due 10:45am on 10/20

**Note:** Answers to the exercises listed below should be given in *typewritten form* (latex formatting preferred) by the deadline mentioned above. You should email your answers and also all the Maude code and all screenshots of your tool interactions for solving Problem 2 to nishant2@illinois.edu.

1. Solve Exercise 100 in *STACS*.

   **Note.** Your equality proofs should *only use the axioms of the theory of groups listed in Exercise 100 in STACS* and no other equations. However, equations already proved using such axioms can also be used as *lemmas* to prove other equations.

2. Consider the following module [available in the Latex version of this homework], specifying an alternative axiomatization of the *theory of groups*, where \( i \) denotes the *inverse* operation \((\_)^{-1}\).

   \[
   \text{set} \ 	ext{include BOOL off} .
   \]

   \[\text{fth \ GROUP is}\]

   \[\text{sort \ Group .}\]

   \[\text{op \ 1 : \ Group .}\]

   \[\text{op \ i : \ Group -> \ Group .}\]

   \[\text{op \ \_\_\_ : \ Group \ Group -> \ Group .}\]

   \[\var \ x \ y \ z : \ Group .\]

   \[\text{eq \ (x * y) * z = x * (y * z) .}\]

   \[\text{eq \ 1 * x = x .}\]

   \[\text{eq \ x * 1 = x .}\]

   \[\text{eq \ x * i(x) = 1 .}\]

   \[\text{eq \ i(x) * x = 1 .}\]

   \[\text{eq \ i(1) = 1 .}\]

   \[\text{eq \ i(i(x)) = x .}\]

   \[\text{eq \ i(x * y) = i(y) * i(x) .}\]

   \[\text{eq \ x * (i(x) * y) = y .}\]

   \[\text{eq \ i(x) * (x * y) = y .}\]

   \[\text{endfth}\]

   Do the following:

   (a) Prove that the module GROUP is locally confluent by using the Church-Rosser Checker tool.

   (b) Use the MTA tool to prove the termination of GROUP.

   **Hint.** This module has a termination proof by an \( A \lor C \)-RPO order.

   (c) Call \( \Sigma_G \) the signature of the theory of groups, \( E_G \) the axiomatization of the theory of groups in Exercise 100 in STACS, and \( E'_G \) the equations in the above theory GROUP. Prove the theory equivalence:

   \[
   (\Sigma_G, E_G) \equiv (\Sigma_G, E'_G)
   \]

   **Hint.** To prove (c) more easily you may find it useful to use Maude to automate part of the proof, taking advantage of the fact that, as you have already proved in parts (a) and (b), \( E'_G \) is confluent and terminating.