cs473 Algorithms

Problem Set #8

Prof. Michael A. Forbes — Due: *Fri., 2024-04-05 17:00*

All problems are of equal value.

1. Consider the linear program Π defined by

$$
\begin{aligned}
& \max 2x + y \\
\text{st} \quad & x + 2y \leq 12, \\
& -x + 3y \leq 9, \\
& 2x - 3y \leq 8, \\
& x, y \geq 0
\end{aligned}
$$

   (a) Define a *vertex* to be a point (x, y) that is both feasible, and makes two of the linear *in*equalities into *equalities*. For example, $(0, 0)$ is a vertex because it is feasible and meets the constraints $x \geq 0$ and $y \geq 0$ with equality.

   Give a list of all vertices of Π, with a proof of correctness.

   (b) Draw P as a subset of $\mathbb{R}^2$.

   (c) Derive the dual $\amalg$ to Π.

   (d) As the dual $\amalg$ is in three variables, a vertex of this system of constraints now asks for *three* of the inequalities to be met. Give a list of all vertices, with a proof of correctness.

   (e) Solve Π by providing a primal feasible point and a dual feasible point witnessing that $|\Pi| = |\amalg|$.

2. Minimum-cost circulation. Erickson Chapter H, Problem #3.

3. Large squares and rectangles. Erickson Chapter H, Problem #4 (a),(b),(d) *only*.