cs473 Algorithms

Problem Set #0

Prof. Michael A. Forbes — Due: *Fri., 2022-01-26 17:00*

Some reminders about logistics. See the course webpage for full details.

- **Submission Policy:** Submit psets via gradescope. Student psets must obey the following constraints:
  - Each problem starts on its own page.
  - The first page has the following metadata:
    - author(s) of the problem set
      - name(s)
      - netid(s)
    - pset number
    - list of collaborators
- **Collaboration Policy:** For *this* problem set, each student must work independently and submit their *own* solutions. For the *remaining* problem sets, students are allowed to work in groups of up to three.
- **Late Policy:** Late psets are not accepted. Instead, several lowest-scoring pset problems will be dropped from a students score.

All problems are of equal value.

1. Solve the following recurrences, by giving an asymptotically tight bound of the form $\Theta(f(n))$ where $f(n)$ is a standard and well-known function. Assume as a base case that $T(n) = 2$ for $n \leq 16$. No proofs are necessary.
   (a) $T(n) = T(n/2) + n^3$.
   (b) $T(n) = 4T(n/4) + n$.
   (c) $T(n) = n^{2/3}T(n^{1/3}) + n$.
   (d) $T(n) = 3T(n^{1/2}) + \log n$.
   (e) $T(n) = 2T(n-5) + 3$.
2. Paths with lengths a multiple of 6. Erickson Chapter 5, Problem #14 (http://jeffe.cs.illinois.edu/teaching/algorithms/book/05-graphs.pdf).
3. A class of m students will be taking an exam in a room. The room has n^3 tables, one on each point in the grid $\{1, \ldots, n\} \times \{1, \ldots, n^2\}$. Each student sits at an independent and uniformly randomly chosen table. The instructor has set up a fan that blows in the x-direction toward infinity (along $(i, 1)$ to (i, n^2) for any i) in order to enhance circulation.
   (a) Compute the expected number of students who sit at a table that is also occupied by another student.

(b) A student is *socially distanced (accounting for the fan)* if they sit at a table (i, j), and no other student sits in any of the tables $\{(i, j), (i-1, j), (i-1, j+1), (i-1, j-1)\}$ (note that these 4 locations may not be locations of actual tables). Compute the expected number of socially distanced students.

(c) Suppose that out of the m students, k are ill. A non-ill student sitting at (i, j) is then *exposed* if, (a) an ill student also sits at (i, j), or (b) at least two ill students sit in the locations $\{(i-1, j), (i-1, j+1), (i-1, j-1)\}$. Compute the expected number of exposed students.