CS 473: Algorithms, Spring 2021
HW 0 (due Wednesday, Feb 3rd at 8pm)

This homework contains three problems. **Read the instructions for submitting homework on the course webpage**.

Collaboration Policy: For this home work, each student should work *independently* and write up their own solutions and submit them.

1. Solve the following recurrences in the sense of giving an asymptotically tight bound of the form $\Theta(f(n))$ where $f(n)$ is a standard and well-known function. No proof necessary for the first four parts; simply state the bound.
   (a) $A(n) = n^{1/3}A(n^{2/3}) + n$, $A(n) = 1$ for $1 \leq n \leq 8$.
   (b) $B(n) = B(n/2) + n$, $B(1) = 1$.
   (c) $C(n) = 2C(n-1) + 1$, $C(1) = 1$.
   (d) $D(n) = 3D(n/3) + 4D(n/4) + n^3$, $D(n) = 1$ for $n \leq 4$.
   (e) *Prove* by induction that the $T(n)$ defined by the recurrence
   $$T(n) = 2T(\sqrt{n}) + \log n$$
   if $n \geq 4$, and $T(n) = 3$ if $n < 4$ satisfies the bound $T(n) = O(\log n \log \log n)$.
2. Problem 18 from Erickson's book chapter `http://jeffe.cs.illinois.edu/teaching/algorithms/book/05-graphs.pdf`
3. Consider the standard balls and bins process. A collection of m identical balls are thrown into n bins: each ball is thrown independently into a bin chosen uniformly at random.
   (a) What is the (precise) probability that a particular bin i contains exactly k balls at the end of the experiment?
   (b) Suppose $m = n$. Let Y be the number of bins that has exactly one ball. What is the expectation of Y?
   (c) What is the variance of Y?

   Explain your calculations when you derive the bounds.