L25: NPC 11/28/23

- NP / Cook theorem. (CSAT)
- CLIQUE $\equiv_p$ INDEP SET $\equiv_p$ VERTEX COVER
- CSAT $\geq_p$ SAT
- SAT $\geq_p$ 3SAT

HER PROBLEMS

SUBSET SUM
PARTITION
HAMILTONIAN CYCLE
3COLORING

<u>SAT $\geq_p$ 3SAT</u>

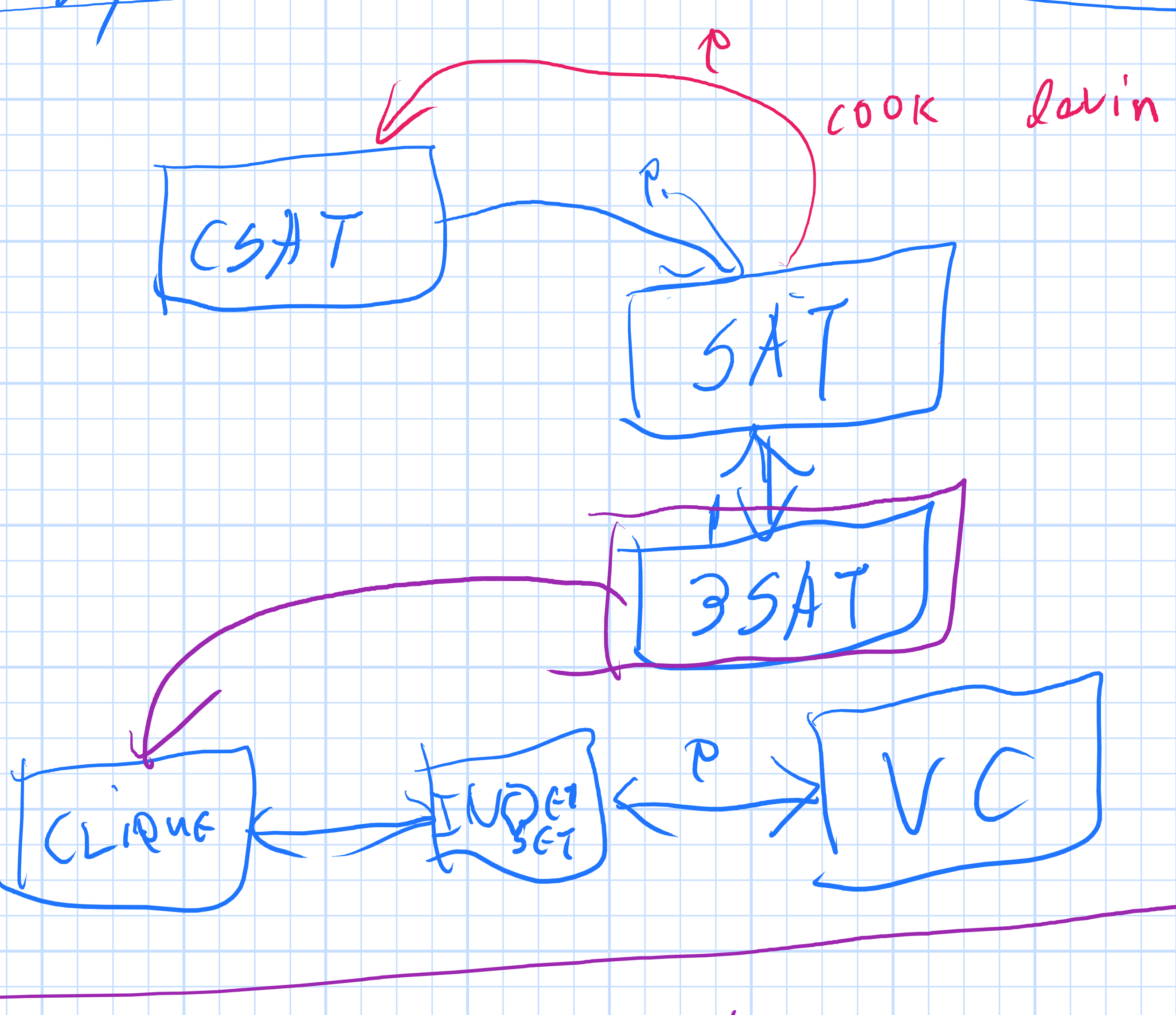
3SAT $\equiv$ SAT for 3CNF formula
$(x \vee y \vee z)$

$x \vee y \Rightarrow (x \vee y \vee z) \wedge (x \vee y \vee \bar{z})$
new variables

$x \Rightarrow (x \vee \bar{\alpha} \vee \bar{\beta}) \wedge (x \vee \bar{\alpha} \vee \beta) \wedge (x \vee \alpha \vee \bar{\beta}) \vee (x \vee \alpha \vee \beta)$

$x \vee y \vee z \vee w \Rightarrow (x \vee y \vee \alpha) \wedge (\bar{\alpha} \vee z \vee w)$

$(x \vee y \vee z)$



CLIQUE IS NP-complete

<u>CLIQUE PROBLEM</u>

Instance: $\langle G, k \rangle$
Q: Does G has clique of size k???

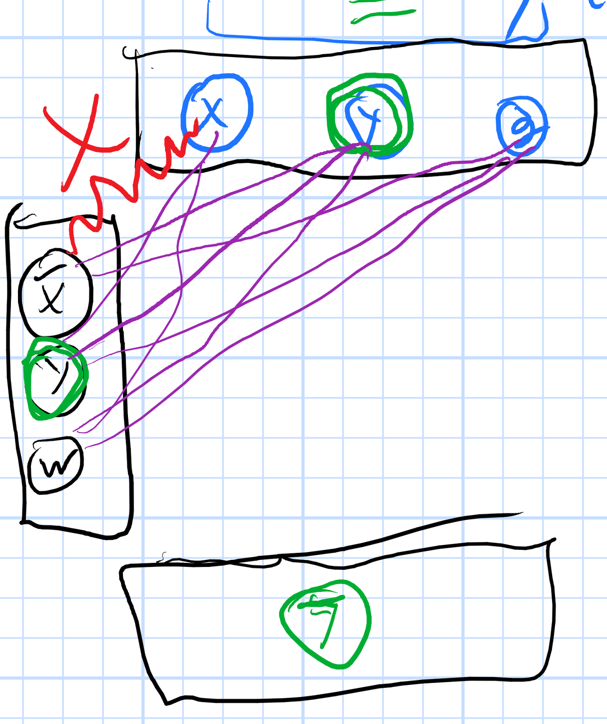
Show reduction from 3SAT to CLIQUE

Input instance: φ a 3SAT formula
output of the reduction: G, and k

φ is satisfiable $\Longleftrightarrow$ G has clique of size k

Write the formula "as" graph

$(x \vee y \vee z) \wedge (\bar{x} \vee y \vee w) \wedge \dots$



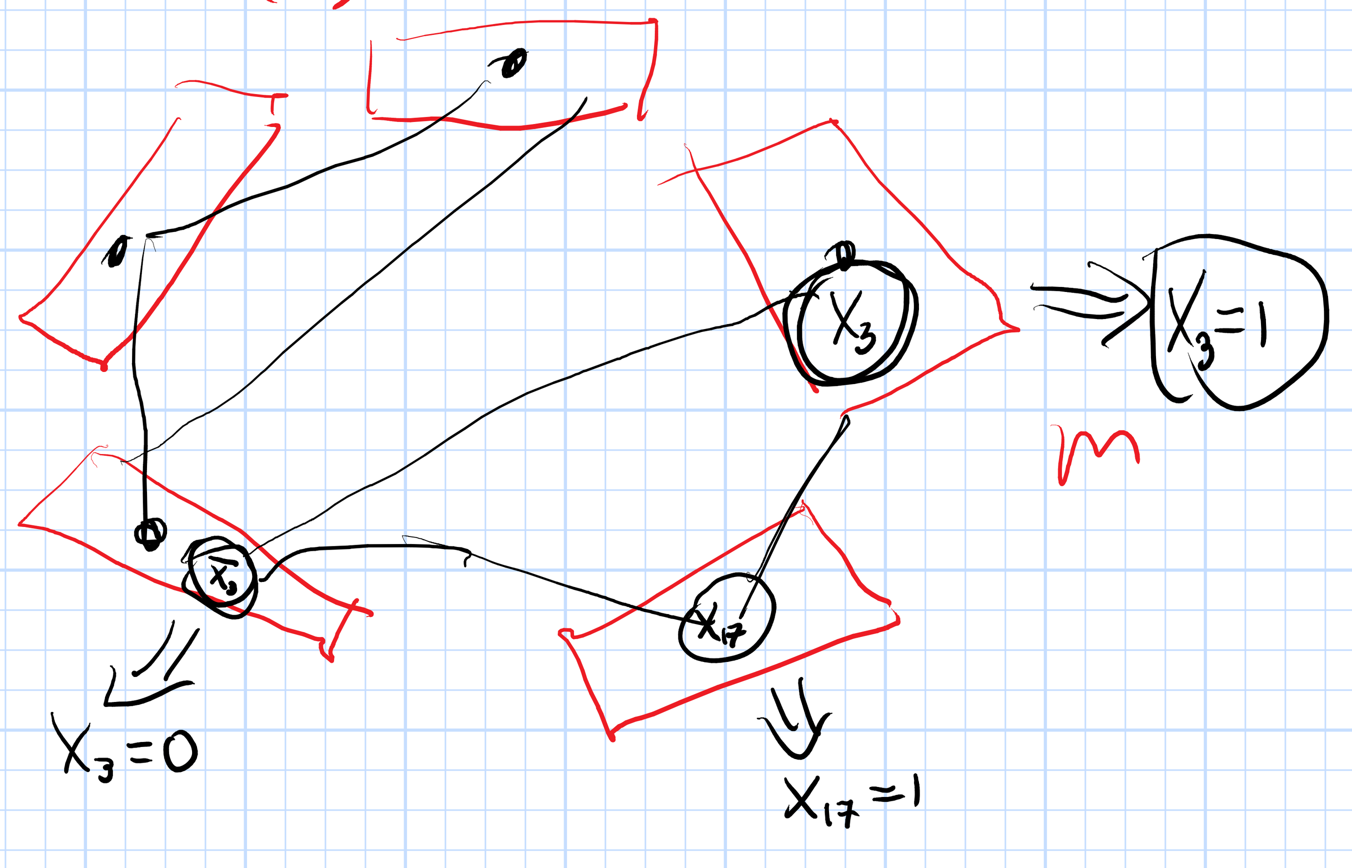
m = # of clauses in φ

G would have 3m vertices

m parts

m clauses
$\Rightarrow$ G has 3m vertices
G has $\leq O(m^2)$.

$\Rightarrow$



<u>Theorem</u> CLIQUE IS NPC.

$\Rightarrow$ Independent Set and Vertex Cover are NPC.

<u>HAMILTONIAN CYCLE</u>

G: Undirected graph
Q: Is there a simple cycle that visits every vertex exactly once?

Reduction from VC to HC.

<u>Proof</u>

$\langle G, k \rangle$

Draw G where every vertex is a horiz line