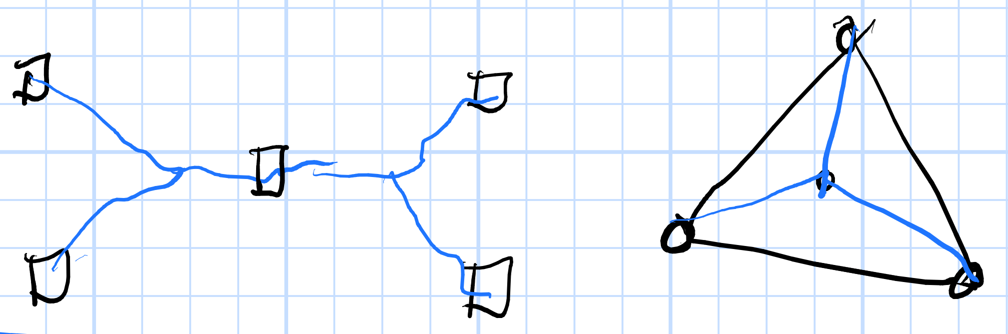
L9 DP + Rand alg 9/19/23

Steiner trees

$G=(V,E)$ undirected, positive weights on the edges.

$X \subseteq V$

Q: Compute cheapest connected subgraph $H \subseteq G$ s.t. $X \subseteq V(H)$

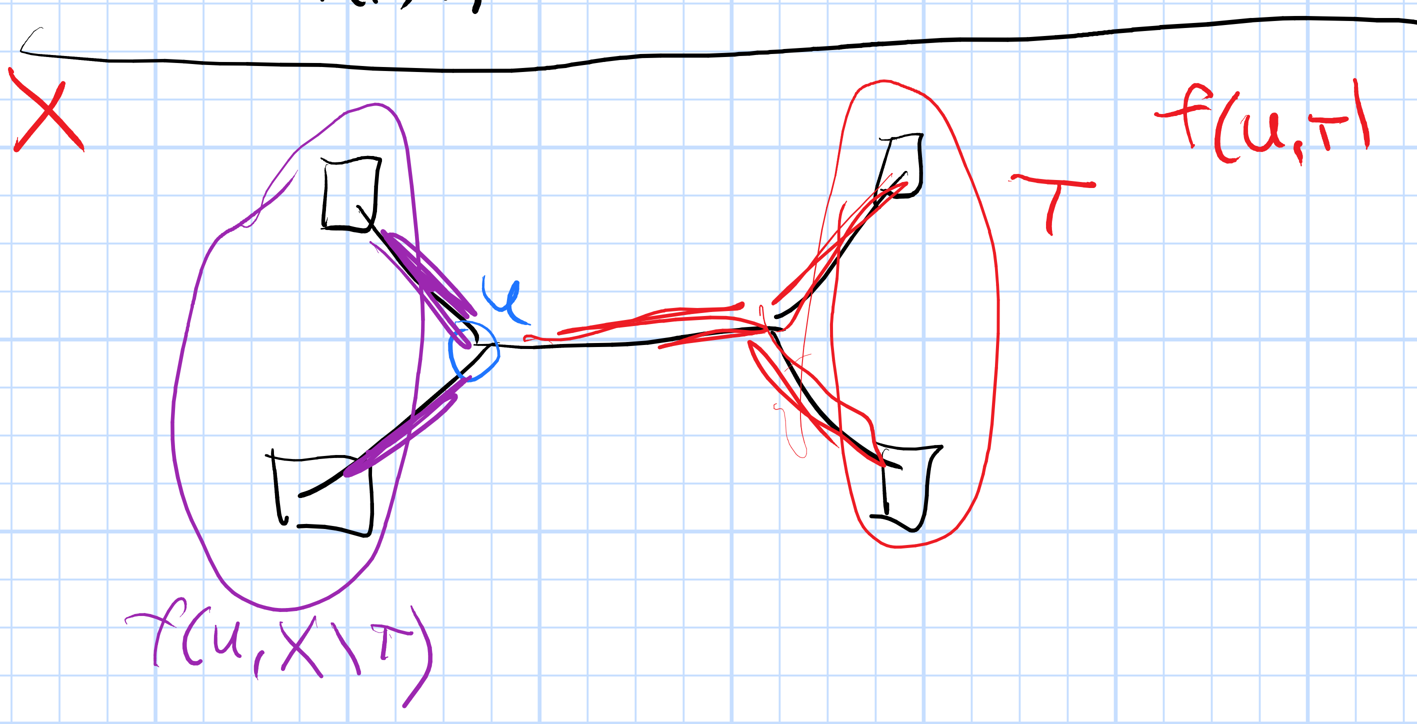


Steiner tree is NP-Hard.

$k = |X|$: parameter of the problem

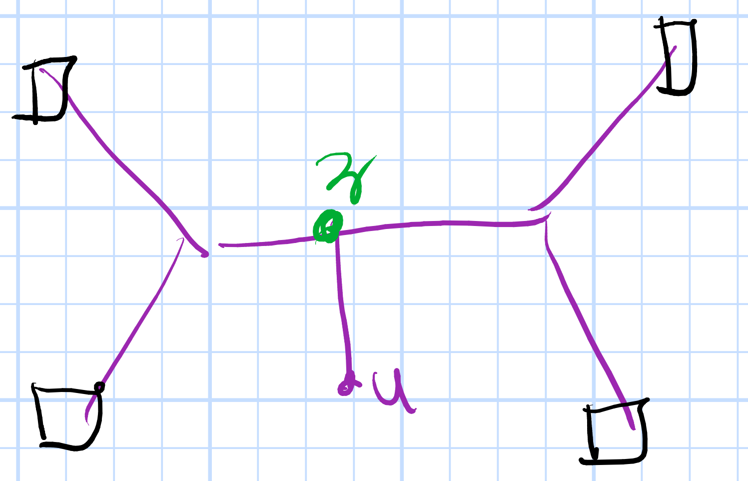
An algorithm A is FPT (fixed parameter tractable) if its running time is

$f(k)\, n^{O(1)}$



$f(u,T) \equiv$ price of the cheapest steiner tree spanning the terminals of T, and ... u.

$$f(u,T) = \begin{cases} 0 & T=\emptyset \\ d_G(u,t) & T=\{t\} \\ f(u, T-u) & u \in T \\ \min\limits_{z \in V} \min\limits_{\substack{S \subseteq T \\ S \neq \emptyset \\ S \neq T}} \left(f(z,S) + f(z, T\setminus S) + d_G(z,u) \right) \end{cases}$$



$X \subseteq V$: $k=|X|$ $n = |V(G)|$

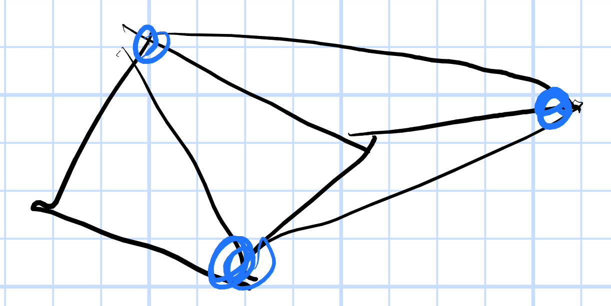
$\underbrace{f(u,T)}_{n \quad 2^k}$

$2^k n$

$O(2^k n)$ time to complete a recursive call ignoring time spent in recursion

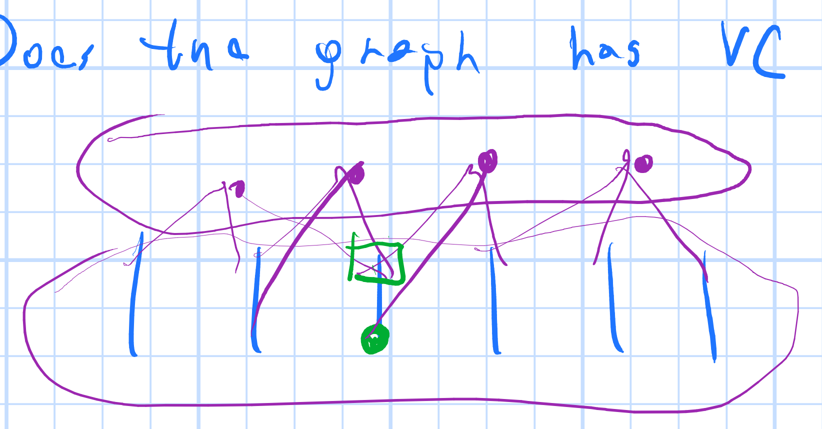
$O(4^k n^2 + n^3)$

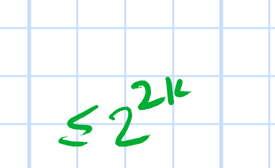
FPT example VC: Vertex cover

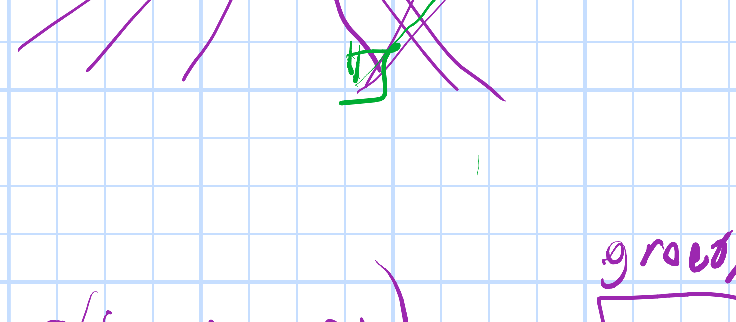


$e_1 \,|\, e_2 \,|\, \dots \,|\, e_t$ $t \le k$

Q: Does the graph has VC of size k?







$\le 2^{2k}$

$O(n+m)\; 2^{2k}$

greedy matching $O(n+m)$